ENHANCEMENT TO THE LUGE MODEL FOR GLOBAL DESCRIPTION OF FRICTION PHENOMENA

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Abstract—The LuGre model of friction—a bristle based model—predicts important friction phenomena useful for control of mechanical systems. This model accurately describes the behavior of control systems for small initial conditions. This paper proposes a simple but fundamental modification of the model in order to improve the reliability in a global sense. This improvement increases the comprehension of experimental evidences in control of mechanisms.

Keywords—Control, Friction, Mechanisms, LuGre model, PD control.

I. INTRODUCTION

The study of friction in the automatic control community has grown during the last decade (Armstrong-Helouvy, 1991; Armstrong-Helouvy, et al. 1994; Alvarez–Ramirez, et al. 1995). The reason is that friction is responsible for many undesirable phenomena observed in implementation of control systems for high precision mechanical systems such as robotic manipulators. Friction produces undesirable behaviors in control systems such as positioning and tracking errors, and limit cycles (Shapiro, 2000). Compensating for friction to attenuate these effects has been one of the main research issues in mechanism control over the years (see Lischinsky et al. (1999) and reference therein).

Viscous friction and Coulomb friction are by far the most popular ingredients in friction models utilized for control of mechanical systems. More elaborate models incorporate in addition to viscous and Coulomb friction also the Stribeck effect, to better capture the behavior of motion at low velocity (Armstrong-Helouvy, 1991). Although these friction models are simple because they establish that friction force or torque depends on the instantaneous relative velocity between the bodies in contact, there exist also dynamic friction models where the actual friction force or torque is a function of the instantaneous velocity but also of the previous behavior (Ludema, 1996). To the latter class belongs the LuGre model proposed in Camadas et al. (1995) which describes the effects of viscous and Coulomb friction but also more complex friction behavior such as stick-slip motion (Polycarpou and Soom, 1995), presliding displacement (Hsieh and Pan, 2000), and Stribeck effect.

The LuGre model for friction proposed in Camadas et al. (1995) seems to accommodate pretty well to expectation of improving precision in mechanisms by control based in such a friction model. Nevertheless, in this paper we present simulation evidences of a simple mechanical system incorporating this friction model which predicts unrealistic behavior when departing from certain initial conditions. To overcome this drawback, this paper proposes a simple but fundamental modification to the model allowing to match its predictions to real expectations irrespective of the initial conditions.

II. LUGE MODEL OF FRICTION

The LuGre model widely described in Camadas et al. (1995) consists of a differential equation where the relative velocity \( \dot{q} \) between the bodies in contact is the system input, and the friction force or torque \( f \) opposing to the bodies motion is the system output. The LuGre model can be written in a suitable form as

\[
\dot{z} = -\frac{|\dot{q}|}{g(\dot{q})} z + \dot{q},
\]

(1)

\[
f = \left[ \sigma_0 - \sigma_1 \frac{|\dot{q}|}{g(\dot{q})} \right] z + |\dot{q} + f_a| \dot{q},
\]

(2)

where

\[
g(\dot{q}) = \frac{f_c + [f_a - f_e] e^{-\sigma_0 (\dot{q})^2}}{\sigma_0} > \frac{f_c}{\sigma_0} > 0,
\]

(3)

\( z \) is an immeasurable state variable called “average deflection of the bristles” (Camadas et al., 1995) because the friction interface between the bodies is thought as a contact between bristles (Rice and Mosleh, 1997), \( f_c \) is the Coulomb friction coefficient, \( f_a \) is the static friction coefficient satisfying \( f_s > f_c \), \( f_e \) is the viscous friction coefficient, and \( v_s \) is the Stribeck velocity coefficient. Finally, \( \sigma_0 \) and \( \sigma_1 \) are the stiffness and damping coefficients respectively. In sum, the LuGre model is composed by the differential equation (1) and the output equation (2).
III. LIMITATION OF THE LUGRE MODEL

In this section we show that the LuGre model fails to predict a specific behavior when departing from certain initial conditions. To illustrate this behavior, let us consider as mechanism an immovable rigid body having inertia $J$ where friction $f$ opposes to the rotational motion:

$$J \ddot{\theta} = \tau - f$$

where $\tau$ is the external applied torque.

Consider that friction $f$ is modeled by the LuGre friction model (1)-(2), and the applied torque $\tau = \tau_0 > 0$ is a constant torque. Hence, the system dynamics is governed by the following nonlinear differential equation:

$$\frac{d}{dt} \left[ \begin{array}{c} q \\ \dot{q} \\ z \end{array} \right] = \left[ \begin{array}{c} \tau_0 - \left[ \sigma_0 - \sigma_1 \left( \frac{\dot{q}}{\sqrt{1 + \frac{\dot{q}^2}{\sigma_1^2}}} \right) \right] \dot{q} - \left[ \sigma_1 + f_s \right] \ddot{q} \\ \frac{1}{\sqrt{1 + \frac{\dot{q}^2}{\sigma_1^2}}} \dot{q} + \ddot{q} \end{array} \right]$$

whose equilibria set is given by

$$E = \left\{ \left[ \begin{array}{c} q \\ \dot{q} \\ z \end{array} \right] \in \mathbb{R}^3 : \dot{q} = 0, z = \frac{\tau_0}{\sigma_0} \right\}.$$  \hspace{1cm} (5)

The equilibrium set $E$ in (6) always exists no matter how large the input torque $\tau_0$ might be. This simple analysis of the equilibrium set $E$ means that if the initial condition for the system originates within any point in the set $E$, i.e. $[q(0) \ \dot{q}(0) \ z(0)]^T \in E$, then the body will remain at rest in its initial position, i.e. $q(t) = q(0)$ for all $t \geq 0$, irrespective of the value of the applied constant torque $\tau_0$. Although the claim is true for small values of the applied torque $\tau_0$, the conclusion is obviously wrong for enough large applied torques $\tau_0$ because motion will be always present, thus the position $q(t)$ cannot remain constant. This incoherence of the LuGre model is at the origin of the enhancement proposed in this paper. Before that, let’s see the system behavior more in details.

Hereafter we pay attention to the particular case when the applied constant torque $\tau_0$ is larger than the static friction coefficient $f_s$, i.e. $\tau_0 > f_s > 0$. Extensive simulations of system (5) have been carried out to capture the qualitative behavior of the system. The results of these simulations are summarized in Fig. 1 where some trajectories in the state space are depicted. Two important structures in the state space are shown in Fig. 1: an invariant set denoted by $IS$, and the equilibrium set $E$. Let us analyze each structure in the following paragraphs.

Figure 1 depicts a structure in the state space corresponding to the invariant set $IS$ defined by

$$IS = \left\{ \left[ \begin{array}{c} q \\ \dot{q} \\ z \end{array} \right] \in \mathbb{R}^3 : \dot{q} = \dot{q}^*, z = g(\dot{q}^*) \text{ sgn} (\dot{q}^*) \right\}.$$ \hspace{1cm} (7)

where $\text{sgn}(\dot{q}^*)$ is the sign “function” defined as

$$\text{sgn}(\dot{q}^*) = \begin{cases} 1 & \text{ if } \dot{q}^* > 0 \\ -1 & \text{ if } \dot{q}^* < 0 \end{cases}$$

and $\dot{q}^*$ is the unique solution $\dot{q}$ of

$$\tau_0 - [f_s + f_s e^{-\frac{\dot{q}^2}{\sigma_1^2}}] \text{ sgn}(\dot{q}) - f_s \dot{q} = 0.$$

From definition (7) and the system model (5), it can be proven that the set $IS$ is an invariant set because for any initial condition in $IS$, the corresponding solution remains within $IS$, i.e.

$$\begin{aligned} [q(0) \ \dot{q}(0) \ z(0)]^T & \in IS \Rightarrow [q(t) \ \dot{q}(t) \ z(t)]^T \in IS \ \forall t \geq 0. \end{aligned}$$

This invariant set $IS$ is expected to be there because experimental evidences demonstrate that if a constant torque $\tau_0$ larger than the static friction coefficient $f_s$ is applied to the body, then it will move and converge asymptotically to a constant velocity $\dot{q}^*$ different from zero.

As previously pointed out, the equilibrium set $E$ appears in the state space depicted in Fig. 1. Some trajectories converge to the set $E$ even though the simulations have been obtained for a large applied torque in the sense that $\tau_0 > f_s$. All the points in the set correspond to zero velocity, that is, all trajectories converging to the set $E$ finally yield $\dot{q}(t) \to 0$ as $t \to \infty$. This prediction of the LuGre model contradicts the expected behavior of the system.

Although the validity of the LuGre is correct in a local region around of the invariant set $IS$, e.g. by restricting the initial condition $z(0)$ to be small in the sense $|z(0)| < f_s/\sigma_0$, we believe that it is important to have a full global model to describe the behavior in the complete state space. This is the motivation to introduce the following improved LuGre model.

IV. IMPROVED LUGRE MODEL

The mathematical explanation to the incorrect prediction generated by the LuGre model when the applied
torque $\tau_0$ is larger than the static friction coefficient $f_s$ is the existence of the equilibria set $E$. Thus, the LuGre model must be adjusted in such a way that the equilibrium set $E$ do not exist any more when $\tau_0 > f_s$.

Towards this end, notice that the equilibria $E$ given in (6) contain $\dot{q} = 0$ and $z$ solution of

$$\tau_0 - \sigma_0 z = 0$$

(8)

which results in $z = \tau_0/\sigma_0$ irrespective of the value of $\tau_0$. Since $\dot{q} = 0$ appears in any possible equilibrium, hence one way to guarantee that no equilibria exist is by modifying (8) in such a way that the solution $z = \tau_0/\sigma_0$ exists if and only if $\tau_0 < f_s$. Let us propose the following simple expression

$$\tau_0 - C \text{sat}(z; f_s/\sigma_0) = 0$$

(9)

where $\text{sat}(x; k)$ stands for the hard saturation function defined as

$$\text{sat}(x; k) = \begin{cases} 
  x & \text{if } |x| \leq k \\
  k & \text{if } x > k \\
  -k & \text{if } x < -k.
\end{cases}$$

for any $x \in \mathbb{R}$ and $k > 0$.

Because $|\text{sat}(z; f_s/\sigma_0)| \leq f_s/\sigma_0$ for all $z \in \mathbb{R}$, then no solution $z$ exists for (9) when $\tau_0 > f_s$ as desired.

These arguments motivate to modify only the output equation (2) of the LuGre model in the following way

$$f = \sigma_0 \text{sat}(z; f_s/\sigma_0) - \sigma_1 \frac{|\dot{q}|}{g(\dot{q})}z + [\sigma_1 + f_s]\dot{q}.$$  

(10)

In sum, the modified LuGre dynamic friction model is described by the state equation (1) and the new output equation (10) with $g(\dot{q})$ still given in (3). As a consequence of the modification, the friction torque output $f$ of the modified LuGre model at rest $\dot{q} = 0$ is now explicitly bounded as $|f| \leq f_s$ which holds for all $q, z \in \mathbb{R}$. This result obviously matches the well known fact that at rest the friction torque is smaller or equal than the static friction parameter.

Let us analyze again the behavior of the simple mechanical system studied previously but using now the modified LuGre friction model (1) and (10). Consider the motor model (4) where the modified LuGre model (1) and (10) is utilized. The motion of the body is governed by

$$\frac{d}{dt} \begin{bmatrix} \dot{q} \\ \dot{z} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{\tau} \left[ \tau_0 - C \text{sat}(z; f_s/\sigma_0) + \sigma_1 \frac{|\dot{q}|}{g(\dot{q})}z - [\sigma_1 + f_s]\dot{q} \right] \\
- \frac{|\dot{q}|}{g(\dot{q})}z + \dot{q} \end{bmatrix}.$$  

(11)

The equilibria of this nonlinear differential equation are those elements of the state space for which $\dot{q} = 0$ and $z$ solution of (9). The latter equation has solution if and only if the applied torque $\tau_0$ is smaller or equal than the static friction coefficient $f_s$. For this case, the equilibria are contained in the set $E$ defined in (6).

Since (9) has no solution $z$ when the applied torque $\tau_0$ is larger than the static friction coefficient $f_s$, then we get the conclusion that the system (11) has no equilibria whereas $\tau_0 > f_s$. This means that the body cannot remain at rest, which agrees with experimental observations and simulation results.

Figure 2: Trajectories in the state space for $\tau_0 > f_s$: Improved LuGre model

A set of numerical simulations for $\tau_0 > f_s$ were conducted of the system (11) derived form the novel improved LuGre model. These simulations include the same initial conditions and parameters that those used to produce Fig. 1. The simulation results depicted on Fig. 2 show that the equilibria set $E$ no longer exist but only the invariant set $\mathcal{IS}$ as desired. The invariant set $\mathcal{IS}$ corresponds exactly to (7) generated from the original LuGre model; this is thanks to the fact that $\text{sat}(g(\dot{q})\text{sign}(\dot{q})); f_s/\sigma_0) = g(\dot{q})\text{sign}(\dot{q})$ because from (3) we have $f_s/\sigma_0 > g(\dot{q})$.

In sum, the proposed improved LuGre model described by (1), (3), and (10), has the following desirable global structural features when $\tau_0 > f_s$:

- The equilibrium set $E$ is not present any more.
- The same invariant set $\mathcal{IS}$ generated by the original LuGre model is preserved.

It is worth remarking that an alternative output equation to (10) is given by

$$f = \sigma_0 \text{sat}(z; f_s/\sigma_0) - \sigma_1 \frac{|\dot{q}|}{g(\dot{q})}\text{sat}(z; f_s/\sigma_0) + [\sigma_1 + f_s]\dot{q}$$

which preserves all the global structural behavior of (10) but also the important physical property of passivity (Barabanov and Ortega, 2000).

V. APPLICATION TO CONTROL A DC MOTOR

The usefulness of the improved LuGre friction model is illustrated in the control of a direct current (DC) motor. A classical linear description of a linear DC motor
considering the torque $\tau$ as the input is given by (4) (Moreno and Kelly, 2002). Consider that the friction $f$ is captured by the new improved LuGre friction model (1), (3) and (10).

Let us consider the Proportional–Derivative (PD) control given by

$$\tau = k_q \ddot{q} - k_v \dot{q}$$  \hspace{1cm} (12)

where $\ddot{q}$ is the positioning error defined as $\ddot{q} = q_d - q$ with $q_d$ being the desired rotor angular position which is assumed to be constant, $k_q$ and $k_v$ are positive gains. It is well known that in the absence of friction, the closed-loop system is globally asymptotically stable (Spong and Vidyasagar, 1989).

The presence of friction may deteriorate the control performance. In order to analyze this situation, let us obtain the closed-loop system by substituting the control action (12) into the motor system composed by (4), (1) and (10). This leads to

$$\frac{d}{dt} \begin{bmatrix} \ddot{q} \\ \dot{q} \\ \dot{z} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{2} k_q \ddot{q} - \sigma_0 \text{sat}(z; \frac{f}{\sigma_0}) + \sigma_1 \frac{m l}{2} \ddot{q} - [k_e + \sigma_1 + f_o] \dot{q} \\ -\dot{q} \\ -\frac{\dot{q}}{\sigma_0} z + \dot{q} \end{bmatrix}$$

which has the equilibrium set depicted in Fig. 3 and characterized by

$$W = \left\{ \begin{bmatrix} \ddot{q} \\ \dot{q} \\ z \end{bmatrix} \in \mathbb{R}^3 : \ddot{q} = 0, \dot{q} = \frac{\sigma_0}{k_p} \text{sat}(z; f_s/\sigma_0) \right\}.$$  

According to this analysis, the corresponding positioning errors $\ddot{q}$ for equilibrium in the set $W$ are bounded as

$$|\ddot{q}| \leq \frac{f_s}{k_p}.$$  \hspace{1cm} (13)

Therefore, the upper bound on the positioning error $|\ddot{q}|$ can be decreased by increasing the controller gain $k_p$ (see Fig. 3). This intuitive behavior which states that the position error due to static friction can be reduced by increasing the gain $k_p$ is predicted by the original LuGre model but under the constraint of $|\ddot{q}(0)| \leq f_s/\sigma_0$ which has been now relaxed thanks to the proposed modification.

**VI. CONCLUSIONS**

A modification to improve the LuGre model of dynamic friction has been proposed. This allows a global description of the friction phenomena, therefore to predict a behavior closer to experimental observations. The modification remains quite simple, and keeps the fine features of the original LuGre model. Finally, the idea behind the modification applies straightforward to other dynamic models of friction such as the Dahl model (Dahl, 1976).
REFERENCES


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