LAGRANGEAN DECOMPOSITION APPLIED TO MULTIPERIOD PLANNING OF PETROLEUM REFINERIES UNDER UNCERTAINTY

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***Abstract*— This work presents a stochastic multiperiod model for representing a petroleum refinery. Uncertainty is taken into account in parameters such as demands, product sale prices and crude oil prices. In the present work, uncertainty is considered as a set of discrete scenarios, each representing a possible shifting of market expectations. Every environment is weighted through an expected probability of occurrence. Previous work revealed that the computational effort of uncertain multiperiod refinery production planning models grows exponentially with the number of time periods and scenarios. Therefore, in order to reduce the computational effort over uncertain long-planning horizons, special techniques must be employed. The proposal is to apply Lagrangean Decomposition, which exploits the block-diagonal structure of the problem, to reduce solution time by decomposing the model on a temporal basis. Solution of the proposed algorithm showed a significant reduction in computational effort with respect to the full-scale outer approximation solver.**

***Keywords*— Lagrangean decomposition, uncertainty, petroleum refinery, planning, MINLP.**

I. INTRODUCTION

Commercial tools that can support the decision making process of production planning of refineries are currently based on linear models that rely on constant yields. This limitation motivated the developed of more accurate representations. One of the first contributions to consider nonlinearity in the production planning is that of Pinto and Moro (2000). According to their proposed framework, every unit is represented as an entity and the complete refinery topology is defined by connecting unit streams. Nonlinearity arises mainly from blending equations and physical properties. Later, Neiro and Pinto (2005) extended the model by accounting for multiple time periods and uncertainty expressed in terms of discrete scenarios.

In order to tackle the large computational effort that results from the size of planning problems, tailored solutions strategies were developed. Ponnambalam et al. (1992) developed an approach that combines the simplex method for linear programming with an interior point method for solving a multiperiod planning model in the oil refinery industry. Neiro and Pinto (2006) developed decomposition methods that are derived from the cross -decomposition theory that prevents the use of master problems. The proposed strategies rely on methods such as Lagrangean decomposition and Lagrangean/surrogate relaxation.

The objective of this paper is to develop efficient solution techniques for multiperiod planning models under uncertainty. The refinery planning model of Neiro and Pinto (2005) with discrete scenarios and corresponding probabilities assigned to the possible market environments is used. The resulting model generates large-scale MINLP problems that are then solved with Lagrangean-based decomposition methods.

II. PROBLEM STATEMENT

The problem to be considered concerns a real-world production planning of the REVAP refinery from Petrobras, located in São José dos Campos (SP, Brazil). A broader discussion on refinery models for planning operations can be found in Pinto *et al.* (2000) and Neiro and Pinto (2005). Generally, it is assumed that in each unit intermediate inlet streams are always mixed and intermediate streams that leave any unit may be sent to several destinations. Therefore, there may be mixing (splitting) before (after) each of the units. The refinery may acquire crude oil from different suppliers that are able to provide petroleum types with different properties and purchase prices. The refinery produces several products that present varied demand profiles and selling prices along a planning horizon that is divided into t discrete time periods of equal duration, $t \in \mathbf{T}$. In addition, uncertainty for petroleum purchase prices, product selling prices as well as product demands are represented through discrete scenarios, $c \in \mathbf{C}$. Each scenario is weighted according to its occurrence probability as detailed in the following section.

III. UNCERTAINTY SCENARIO REPRESENTATION

The main goal of a model that considers uncertainty is to provide a forecast to the planner of how the refinery should perform under several possible scenarios that result from different values of the stochastic parameters. Moreover, it should be noted that solutions do not change depending on the distribution of the probabilities assumed for each scenario. Table 1 shows solutions of an illustrative example in terms of the feedstock selection for three different problems considering a single time period. The first problem considers that a single scenario is feasible, in the second problem two scenarios are possible with the following probability distribution: scenario c_1 with $prob_1$ = 40% and scenario c_2, with $prob_2$ = 60%. Finally, in the third problem it is considered that two scenarios are possible with the following probability distribution: scenario c_1 with $prob_1$ = 60% and scenario c_2 with $prob_2$ = 40%. Problems 2 and 3 have the same scenario c_1 as the only scenario for Problem 1, and scenarios c_2 are also the same in Problems 2 and 3.

Table 1 shows that the solution for scenario c_1 is the same regardless of the problem and the same behavior is observed for scenario c_2. Solutions for the same scenario do not change because the constraints are the same and the only difference concerns the probability parameter of the revenue and costs terms in the objective function. The objective function value should change since each scenario contributes with different probability. So, the multiscenario model returns a weighted objective function based on the probability at which each scenario occurs. More importantly, however, is that the optimization model satisfies the constraints under all different scenarios, as expressed in the model of section 3, which renders a conservative approach for handling uncertainty.

Table 1 – Feedstock selection for three problems considering different scenarios

Crude types	Problem 1	Problem 2		Problem 3	
	c_1 ($prob_1$ = 1.0)	c_1 ($prob_1$ = 0.4)	c_2 ($prob_2$ = 0.6)	c_1 ($prob_1$ = 0.6)	c_2 ($prob_2$ = 0.4)
Marlin	0	0	3758	0	3758
RGN	571	571	2160	571	2160
Cabiun	4417	4417	17993	4417	17993
Albaco	20000	20000	0	20000	0
Condoso	1458	1458	2428	1458	2428

IV. MATHEMATICAL MODEL

The following notation is used in the mathematical model:

Indices:

- c scenario
- p property
- s stream
- t time period
- u, u' unit
- v operating variable

Sets:

- $\mathbf{C}$ scenarios { c | c = 1,…,NC }
- $\mathbf{PI}_u$ properties of the inlet stream of unit u
- $\mathbf{PO}_{u,s}$ properties of outlet stream s of unit u
- $\mathbf{SO}_u$ outlet streams of unit u
- $\mathbf{T}$ time periods { t | t = 1,…,NT }
- $\mathbf{U}$ units of the refinery complex
- $\mathbf{U}_f$ petroleum tanks
- $\mathbf{U}_{feed}$ units that process petroleum
- $\mathbf{UI}_u$ units whose outlet streams feed unit u
- $\mathbf{UO}_{u,s}$ units that are fed by stream s of unit u
- $\mathbf{U}_p$ product tanks
- $\mathbf{US}_{\boldsymbol{u}}$ ordered pair (u',s) that feeds u
- $\mathbf{VO}_u$ operating variables of unit u

Parameters:

- Cb_u pumping cost for unit u
- $Cf_{u,t,c}$ price of petroleum u at t and under c
- $Cinv_{u,t,c}$ inventory cost of product u at t and under c
- $Cp_{u,t,c}$ price of product u at t and under c
- Cr_u fixed operating cost of unit u
- $Cv_{u,v}$ variable cost for operating variable v of u
- $Dem_{u,t,c}$ demand of u at t ($u \in \mathbf{U}_p$) under c
- $PF^L_{u,t}$ LB of inlet property p of unit u
- $PF^U_{u,t}$ UB of inlet property p of unit u
- $prob_{t,c}$ probability of scenario c at time period t
- $Prop_{u,s,p}$ static property p of outlet stream s from u
- QF^L_u LB for feed flow rate of unit u
- QF^U_u UB for feed flow rate of unit u
- $Qgain_{u,s}$ flow rate gain of outlet stream s of unit u
- $Q^L_{u,c}$ LB for outlet flow rate of unit u under scenario c
- QS^L_u LB for outlet flow rate of unit u
- QS^U_u UB for outlet flow rate of unit u
- $Q^U_{u,c}$ UB for outlet flow rate of unit u under scenario c
- $V^L_{u,v}$ LB for operating variable v of unit u
- Vol_u^{Max} storing capacity of tank u
- $V^U_{u,v}$ UB for operating variable v of unit u

Variables:

- $PF_{u,p,t,c}$ property p of the feed stream of unit u at time period t under scenario c
- $PS_{u,s,p,t,c}$ property p of the outlet stream s at unit u at time period t under scenario c
- $QF_{u,t,c}$ feed flow rate of unit u at time period t

under scenario c

$QS_{u,s,t,c}$ outlet flow rate of stream s at unit u at time period t under scenario c

$Q_{u,s,u',t,c}$ flow rate of stream s between units u' and u in time period t for scenario c

$Vol_{u,t,c}$ inventory level of u at time period t under scenario c

$V_{u,v,t,c}$ operating variable v of unit u in time period t under scenario c

$y_{u,t,c}$ binary variable that is 1 if petroleum u ($u \in \mathbf{U}_f$) is chosen at t under scenario c; 0, else.

The problem is denoted **RMP** (Refinery Multiperiod Planning) and is defined as follows:

$$Max\ z = \sum_{c\in\mathbf{C}}\sum_{t\in\mathbf{T}}\left(\sum_{u\in\mathbf{U}_p} prob_{c,t} Cp_{u,t,c}\left(QF_{u,t,c} - Vol_{u,t,c}\right)\right)$$
$$-\sum_{c\in\mathbf{C}}\sum_{t\in\mathbf{T}}\left(\sum_{u\in\mathbf{U}_f}\sum_{s\in\mathbf{SO}_u} prob_{c,t} Cf_{u,t,c} QS_{u,s,t,c}\right)$$
$$-\sum_{c\in\mathbf{C}}\sum_{t\in\mathbf{T}}\left(\sum_{u\in\mathbf{U}_f} Cb_u y_{u,t,c}\right)$$
$$-\sum_{c\in\mathbf{C}}\sum_{t\in\mathbf{T}}\left(\sum_{u\in\mathbf{U}\setminus\{\mathbf{U}_f,\mathbf{U}_p\}}[Cr_u + \sum_{v\in\mathbf{VO}_u}(Cv_{u,v}.V_{u,v,t,c})]QF_{u,t,c}\right)$$
$$-\sum_{t\in\mathbf{T}}\left(\sum_{u\in\mathbf{U}_p} Cinv_{u,t,c} Vol_{u,t,c}\right) \quad (1)$$

Subject to:

Constraints on process units:

$$QF_{u,t,c} = \sum_{(u',s)\in\mathbf{US}_u} Q_{u',s,u,t,c} \quad \forall\ u\in\mathbf{U}\setminus\mathbf{U}_f, t\in\mathbf{T}, c\in\mathbf{C} \quad (2)$$

$$QS_{u,s,t,c} = QF_{u,t,c}.f_{u,s}\left(PF_{u,p,t,c}\right) + \sum_{v\in\mathbf{VO}_u} QGain_{u,s,v}.V_{u,v,t,c}$$
$$\forall u\in\mathbf{U}\setminus\{\mathbf{U}_p,\mathbf{U}_f\}, s\in\mathbf{SO}_u, p\in\mathbf{PI}_u, t\in\mathbf{T}, c\in\mathbf{C} \quad (3)$$

$$QS_{u,s,t,c} = \sum_{u'\in\mathbf{UO}_{u,s}} Q_{u,s,u',t,c}$$
$$\forall u\in\mathbf{U}\setminus\mathbf{U}_p,\ s\in\mathbf{SO}_u, t\in\mathbf{T}, c\in\mathbf{C} \quad (4)$$

$$PF_{u,p,t,c} = \frac{\sum_{(u',s)\in\mathbf{US}_u} Q_{u',s,u,t,c}.PS_{u',s,p,t,c}}{\sum_{(u',s)\in\mathbf{US}_u} Q_{u',s,u,t,c}}$$
$$\forall u\in\mathbf{U}\setminus\mathbf{U}_f, p\in\mathbf{PI}_u, t\in\mathbf{T}, c\in\mathbf{C} \quad (5)$$

$$PS_{u,s,p,t,c} = f_{u,s,p}(PF_{u,p,t,c} \mid p\in\mathbf{PI}_u, V_{u,v,t,c} \mid v\in\mathbf{VO}_u)$$
$$\forall u\in\mathbf{U}\setminus\mathbf{U}_p, s\in\mathbf{SO}_u, p\in\mathbf{PO}_{u,s}, t\in\mathbf{T}, c\in\mathbf{C} \quad (6)$$

Production balance:

$$Vol_{u,t,c} = Vol_{u,t-1,c} + QF_{u,t,c} - Dem_{u,t,c}$$
$$\forall u\in\mathbf{U}_p, t\in\mathbf{T}, c\in\mathbf{C} \quad (7)$$

Petroleum supply constraint:

$$y_{u,t,c}.QS_u^L \le QS_{u,s,t,c} \le QS_u^U.y_{u,t,c}$$
$$\forall u\in\mathbf{U}_f, s\in\mathbf{SO}_u, t\in\mathbf{T}, c\in\mathbf{C} \quad (8)$$

Operation and product quality specifications:

$$QF_u^L \le QF_{u,t,c} \le QF_u^U$$
$$\forall u\in\mathbf{U}\setminus\mathbf{U}_f, t\in\mathbf{T}, c\in\mathbf{C} \quad (9)$$

$$PF_{u,p}^L \le PF_{u,p,t,c} \le PF_{u,p}^U$$
$$\forall u\in\mathbf{U}\setminus\mathbf{U}_f, p\in\mathbf{PI}_u, t\in\mathbf{T}, c\in\mathbf{C} \quad (10)$$

$$V_{u,v}^L \le V_{u,v,t,c} \le V_{u,v}^U$$
$$\forall u\in\mathbf{U}\setminus\{\mathbf{U}_f,\mathbf{U}_p\}, v\in\mathbf{VO}_u, t\in\mathbf{T}, c\in\mathbf{C} \quad (11)$$

$$QF, QS, Q, Vol \in \Re^+;\ PF, PS, V\in\Re;\ y\in\{0,1\} \quad (12)$$

The objective function (1) is defined as the maximization of the revenue obtained by the product sales minus costs related to raw material and operation. The operating cost is a non-linear term that depends on the operating mode of the unit and on the flow rate of the inlet stream. If the unit is operated at its design condition, a base cost that is proportional to the feed flow rate is incurred. Moreover, a proportional cost is incurred, which depends on the value of the deviation variable.

Equation (2) describes mass balances at inlet of unit u. Equation (3) denotes the relationship of the product flow rates with the feed flow rate ($QF_{u,t}$), feed properties ($PF_{u,p,t}$) and operating variables ($V_{u,v,t}$) at each time period t. Equation (3) is valid for units whose product yields closely depend on the petroleum types, such as atmospheric and vacuum distillation columns. Other units usually operate at constant yields; this implies that the variable $PF_{u,p,t}$ is replaced by a corresponding constant parameter. Therefore, Eq. (3) becomes linear for these cases. Equation (4) represents the mass balance at the outlet of unit u. Equation (5) represents a weighted average that relates properties of the unit feed stream with properties of the inlet streams. There are some cases for which properties must be replaced by mixing indices in order to apply Eq. (5) and some properties must be weighted on a mass basis. In the latter cases, the density of the corresponding stream must multiply every term in the numerator and denominator of Eq. (5). Specific examples of Eqs. (3) and (5) are shown in Neiro and Pinto (2004). Equation (6) shows the general relationship among outlet properties, feed properties and operating variables. The functional form of Eq. (6) depends on the unit, stream and property under consideration. Most of the outlet properties are considered constant values, and therefore only a few are estimated. Those are usually properties that depend on petroleum types, such as sulfur content.

Equation (7) represents the inventory level for product tanks at every time period. Equation (8) bounds outlet flow rate for petroleum tanks that are selected; note that there are binary variables $y_{u,t}$ that correspond to the choice of petroleum type u at time period t in order to avoid that insignificant amounts of crude oil are selected (‘tea spoons”). Equation (9) refers to unit capacities, whereas Eq. (10) refers to the properties for product tanks. Equation (11) specifies the operating variable range and Eq.(12) defines domain for the optimization variables. It is important to note that the constraints are defined for each time period t and scenario c, and the objective function (1) maximizes profit under all these time periods and scenarios.

Problem RMP is a Mixed Integer Nonlinear Programming (MINLP) model whose main decisions concern the selection of petroleum types to be processed by the refinery at each time period as well as the amount selected for each of them, the processing units operating plan and inventory management of final products along the planning horizon. Examples of application of the planning model in real-world refineries are presented in Neiro and Pinto (2004, 2005).

V. DECOMPOSITION STRATEGIES

Neiro and Pinto (2005) have solved Problem **RMP** up to 20 time periods and up to 5 scenarios. Results in Figure 1 show an exponential increase in solution time with the number of time periods, as well as with the number of scenarios. Therefore, in order to solve problems for larger number of time periods and scenarios in reasonable solution time, it is necessary to develop a more efficient solution approach.

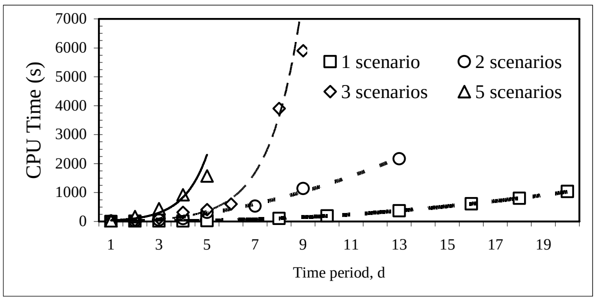

Fig. 1 – Solution time versus number of time periods and scenarios (Neiro and Pinto, 2005)

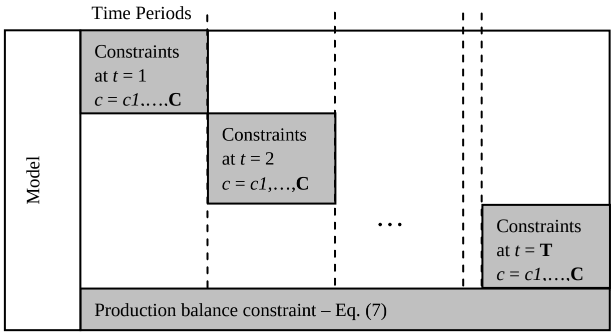

Fig. 2 – Structure of model **RMP**

Because Problem **RMP** presents a block structure (see Fig. 2), Lagrangean Decomposition is a suitable approach that can be applied in order to reduce solution time by decomposing and solving smaller problems with respect to the original problem (Guignard and Kim, 1987).

Neiro and Pinto (2006) and Neiro (2004) have presented several different approaches for solving multiperiod planning problems using Lagrangean Decomposition. In this work the idea is to apply similar ideas in which the planning horizon of **T** time periods is decomposed in **T** problems and solved independently. Figure 3 shows a diagram of the general decomposition steps. The algorithm is initialized with a set of Lagrange multipliers; the dual subproblem ($\mathbf{SD}_\lambda$) is then solved and provides an upper bound on the full-scale problem and y for the primal subproblem ($\mathbf{SP}_y$). The solution of the primal subproblem provides a lower bound (maximization) for the full-scale problem and the Lagrangean multipliers, λ, that are obtained for fixed primal variables, for the dual subproblem. Concerning the convergence test, it checks for bound-improvement and it is based on the observation that solutions of consecutive iterations generated by each of the subproblems are always different, unless the optimal solution is reached. As a result, cycling is prohibited and the algorithm has finite convergence (Van Roy, 1983).

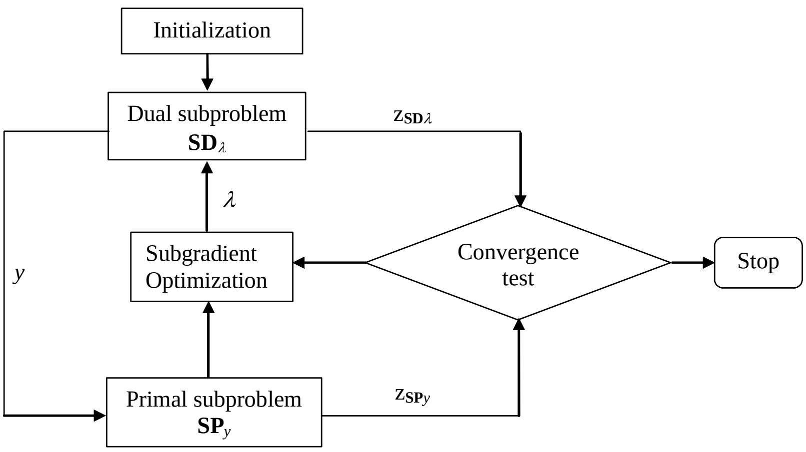

Fig. 3 – Modified Cross Decomposition Method

A set of different combinations of primal-dual subproblems can be used in the decomposition approach of Fig. 3. In this work, two strategies were tested for the solution of the multiperiod production planning for the single refinery model described in section IV. These basically differ in the way the primal subproblems are solved. Sub-sections A and B show the dual and primal subproblems of **RMP** respectively adopted in the two proposed strategies.

A. Dual Subproblem of RMP

Equation (7) can be rewritten as follows:

$$Vol^A_{u,t,c} = Vol^B_{u,t-1,c} + QF_{u,t,c} - Dem_{u,t,c}$$
$$\forall u \in \mathbf{U}_p, t \in \mathbf{T}, c \in \mathbf{C} \qquad (13)$$

$$Vol^A_{u,t,c} = Vol^B_{u,t,c}, \forall u \in \mathbf{U}_p, t \in \mathbf{T}, c \in \mathbf{C} \qquad (14)$$

Now, dualizing constraint (14) yields the following objective function:

$$\begin{aligned}
&Max\\
&z_{\mathbf{RMP\text{-}LR}} = \sum_{c\in\mathbf{C}}\sum_{t\in\mathbf{T}}\left(\sum_{u\in\mathbf{U}_p} prob_{c,t} Cp_{u,t,c}\left(QF_{u,t,c} - Vol^A_{u,t,c}\right)\right)\\
&-\sum_{c\in\mathbf{C}}\sum_{t\in\mathbf{T}}\left(\sum_{u\in\mathbf{U}_f}\sum_{s\in\mathbf{SO}_u} prob_{c,t} Cf_{u,t,c} QS_{u,s,t,c}\right)\\
&-\sum_{c\in\mathbf{C}}\sum_{t\in\mathbf{T}}\left(\sum_{u\in\mathbf{U}_f} Cb_u y_{u,t,c}\right)\\
&-\sum_{c\in\mathbf{C}}\sum_{t\in\mathbf{T}}\left(\sum_{u\in\mathbf{U}\backslash\{\mathbf{U}_f,\mathbf{U}_p\}} [Cr_u + \sum_{v\in\mathbf{VO}_u}(Cv_{u,v}.V_{u,v,t,c})]QF_{u,t,c}\right)\\
&-\sum_{t\in\mathbf{T}}\left(\sum_{u\in\mathbf{U}_p} Cinv_{u,t,c} Vol^A_{u,t,c}\right)\\
&+\sum_{c\in\mathbf{C}}\sum_{t\in\mathbf{T}}\left(\sum_{u\in\mathbf{U}} \lambda_{u,t,c}\left(Vol^B_{u,t,c} - Vol^A_{u,t,c}\right)\right) \qquad (15)
\end{aligned}$$

where $\lambda_{u,t,c}$ are the Lagrangean multipliers for constraint (14). Observe also that the inventory variable $Vol^A_{u,t,c}$ is used in the first and fifth terms. This procedure leads to the following subproblem for a given time period t and scenario c:

Subproblem $\mathbf{RMPLR}_{t,c}$

$$\begin{aligned}
&Max\, z^{t,c}_{\mathbf{RMP\text{-}LRS}} = \sum_{u\in\mathbf{U}_p} prob_{t,c} Cp_{u,t,c}\left(QF_{u,t,c} - Vol^A_{u,t,c}\right)\\
&-\sum_{u\in\mathbf{U}_f}\sum_{s\in\mathbf{SO}_u} prob_{t,c} Cf_{u,t,c} QS_{u,s,t,c} - \sum_{u\in\mathbf{U}_f} Cb_u y_{u,t,c}\\
&-\sum_{u\in\mathbf{U}\backslash\{\mathbf{U}_f,\mathbf{U}_p\}} [Cr_u + \sum_{v\in\mathbf{VO}_u}(Cv_{u,v}.V_{u,v,t,c})]\ QF_{u,t,c}\\
&-\sum_{u\in\mathbf{U}_p} Cinv_{u,t,c} Vol^A_{u,t,c}\\
&+\sum_{u\in\mathbf{U}_p}\left(\lambda_{u,t-1,c} Vol^B_{u,t-1,c} - \lambda_{u,t,c} Vol^A_{u,t,c}\right) \qquad (16)
\end{aligned}$$

Subject to:
Constraints on process units: Eqs (2-6)
Production balance: Eq (13)
Petroleum supply constraint: Eq (8)
Operation and product quality specifications: Eq (9-12)
For every time period t and scenario c

Constraints in subproblem $\mathbf{RMPLR}_{t,c}$ are defined similarly to the ones from Section IV; note however that these constraints are solved independently for each time period and scenario. In other words, the problem for each time period and scenario contains only its corresponding set of constraints and therefore these are not indexed in t and c. An upper bound to the optimal solution of (1) is then given as the sum of the optimal objective values $z^{t,c}_{\mathbf{RMP\text{-}LRS}}$ that are obtained with each subproblem $\mathbf{RMPLR}_{t,c}$ over the planning horizon.

$$\begin{aligned}
&Max\, z_{\mathbf{RMP\text{-}LR}} = Max \sum_{t\in\mathbf{T}}\sum_{c\in\mathbf{C}} z^{t,c}_{\mathbf{RMP\text{-}LRS}}\\
&+Max \sum_{u\in\mathbf{U}_p}\sum_{c\in\mathbf{C}} \lambda_{u,T,c} Vol^B_{u,T,c} \qquad (17)
\end{aligned}$$

where T denotes the last time period. However, it may be noted that the duplicated variable $Vol^B_{u,T,c}$ only appears on the objective function and therefore would be unbounded; thus, it is substituted by its copy that is bounded at T-1. Hence

$$z_{\mathbf{RMP\text{-}LR}} = \sum_{t\in\mathbf{T}}\sum_{c\in\mathbf{C}} z^{t,c}_{\mathbf{RMP\text{-}LRS}} + \sum_{u\in\mathbf{U}_p}\sum_{c\in\mathbf{C}} \lambda_{u,T,c} Vol^A_{u,T,c} \qquad (19)$$

B. Primal Subproblems of RMP

Concerning Problem **RMP**, if the inventory variables are fixed at $Vol^F_{u,t,c}$, connection among time periods is also eliminated. Therefore |**T**| independent subproblems can be defined for every time period t and scenario c as follows:

Subproblem $\mathbf{RMPS}_{t,c}$

$$\begin{aligned}
&Max\, z^{t,c}_{\mathbf{RMPS}} = \sum_{u\in\mathbf{U}_p} prob_{t,c} Cp_{u,t,c}\left(QF_{u,t,c} - Vol^F_{u,t,c}\right)\\
&-\sum_{u\in\mathbf{U}_f}\sum_{s\in\mathbf{SO}_u} prob_{t,c} Cf_{u,t,c} QS_{u,s,t,c} - \sum_{u\in\mathbf{U}_f} Cb_u y_{u,t,c}\\
&-\sum_{u\in\mathbf{U}\backslash\{\mathbf{U}_f,\mathbf{U}_p\}} [Cr_u + \sum_{v\in\mathbf{VO}_u}(Cv_{u,v}.V_{u,v,t,c})]\ QF_{u,t,c}\\
&-\sum_{u\in\mathbf{U}_p} Cinv_{u,t,c} Vol^F_{u,t,c} \qquad (20)
\end{aligned}$$

Subject to
Eqs. (2-12)

In (20), $Vol^F_{u,t,c}$ denotes a fixed value for the inventory variables. The objective function given by (1) is then equivalent to:

$$z_{\mathbf{RMP}} = \sum_{t\in\mathbf{T}}\sum_{c\in\mathbf{C}} z^{t,c}_{\mathbf{RMPS}} \qquad (21)$$

Another possible primal subproblem is obtained by solving problem **RMP** with fixed the binary variables to the values obtained with the solution of the dual subproblem presented in section A.

VI. PROPOSED STRATEGIES

Fig. 4 presents the modified cross decomposition method of Fig 3 applying the dual and primal subproblems described in the previous section. In this figure, ZUB^k and ZLB^k represent upper and lower bounds in iteration k, respectively, whereas UB and LB represent global upper and lower bounds, respectively. Other symbols are: The main difference between ***Strategy* 1** and ***Strategy* 2** is that instead of fixing the binary variables in the primal subproblem as in Strategy **1**, the inventory variables ($Vol_{u,t,c}$) are fixed at the values obtained by the dual subproblem.

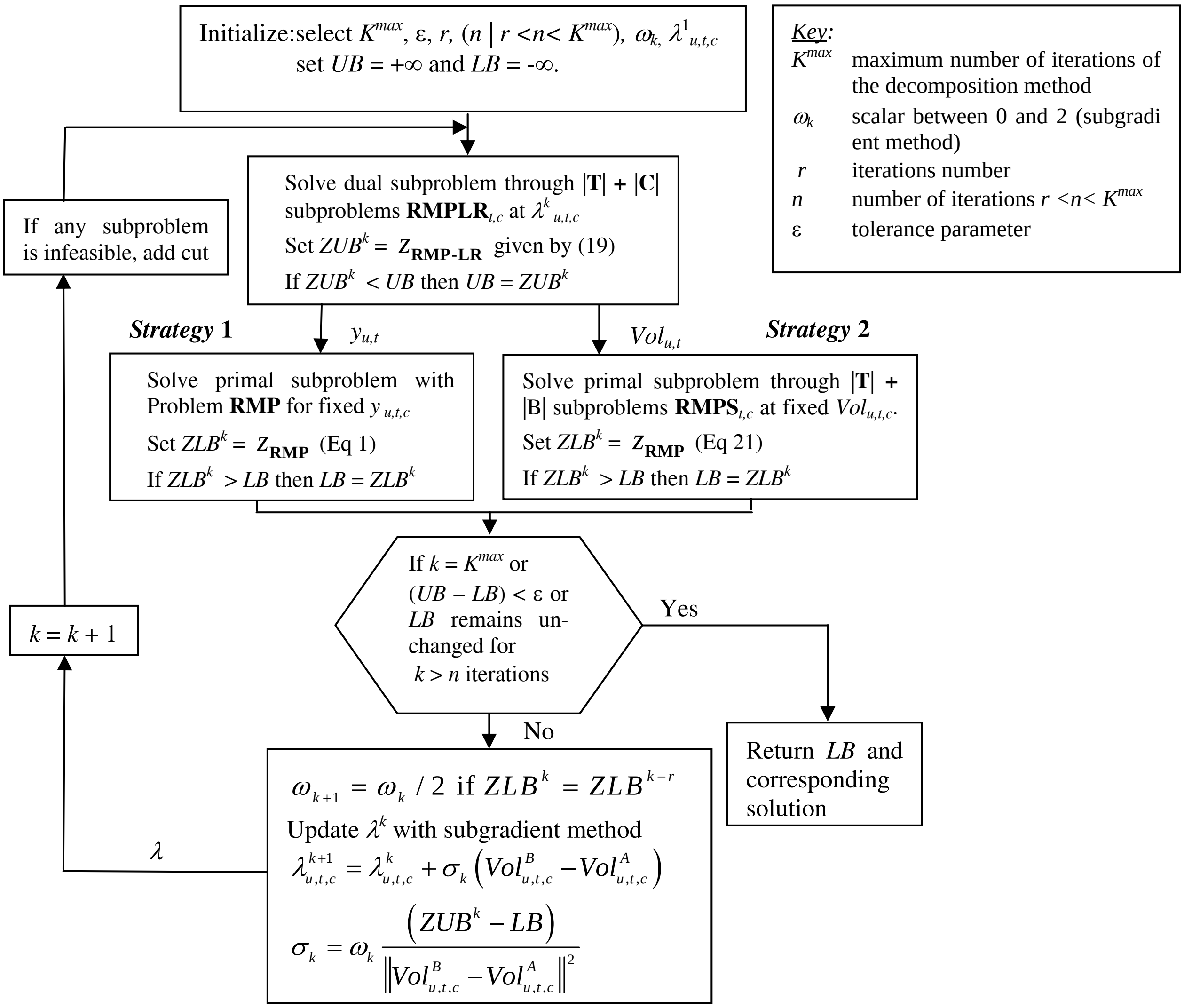

Fig. 4 – Studied strategies

VII. RESULTS AND DISCUSSION

Strategies described in the previous section were used to solve the production planning problem for the REVAP refinery with up to 10 time periods and 5 scenarios. The same problem was solved in Neiro and Pinto (2005) without the use of Lagrangean decomposition. Figure 5 shows the computational solution time of the full-scale Problem **RMP** and of the two proposed strategies. All models and solution algorithms were coded in the GAMS (Brooke *et al.*, 1998) modeling environment. DICOPT++(Viswanathan and Grossmann, 1990) was used to solve the MINLP problems. The NLP subproblems were solved using CONOPT2 (Drud, 1994) and the MILP subproblems were solved with OSL (IBM, 1991) on a PC, Pentium M / 1.6 MHz platform. Table 2 shows the increase in problem size in terms of the number of constraints, continuous variables, binary variables and solution time for the full-scale problem with the increase of the number of time periods and 5 scenarios, which represents the largest instance. Figure 6 shows the comparison of the objective function value obtained through the proposed strategies from that obtained for Problem **RMP** using DICOPT++.

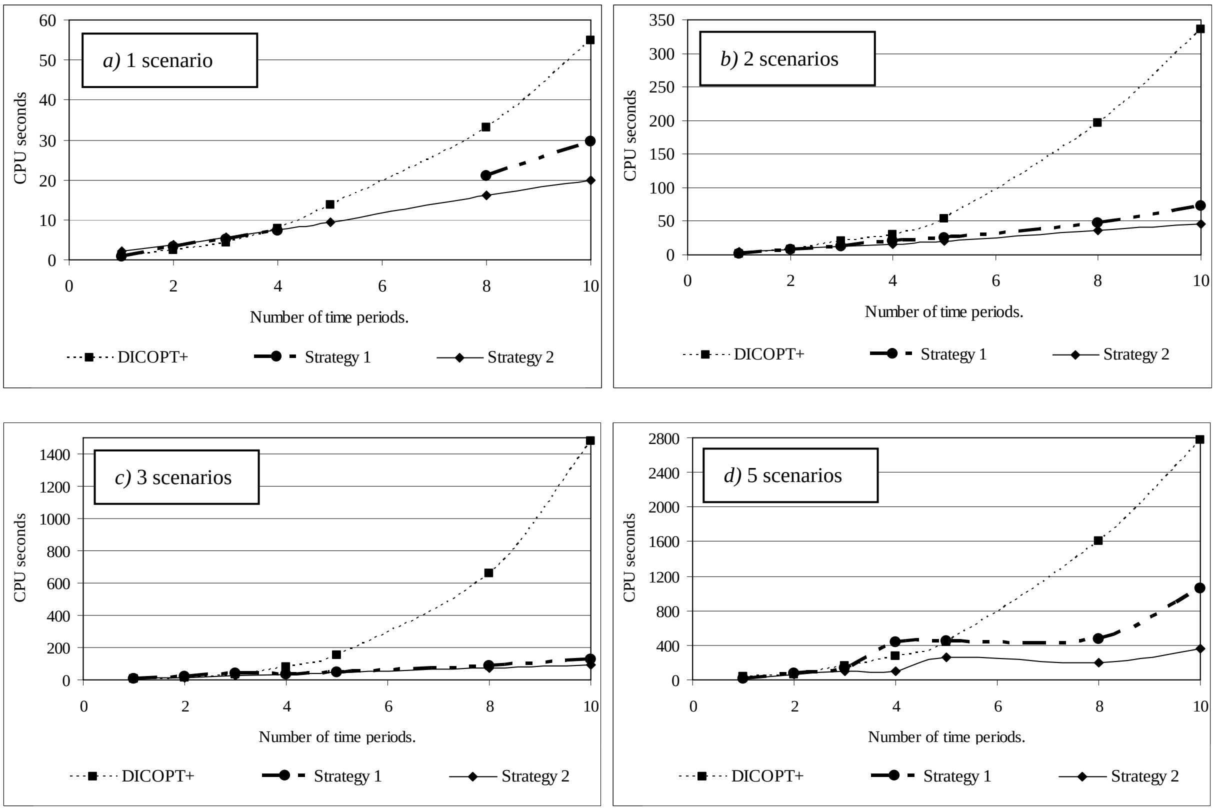

Fig. 5 – Solution time results for ***Strategies 1*** and ***2*** and DICOPT++

Table 2 – Statistics of Problem **RMP** as function of the number of time period and 5 scenarios

	Number of time periods									
	1	2	3	4	5	6	7	8	9	10
variables	1,586	3,171	4,756	6,341	7,926	9,511	11,096	12,681	14,266	15,851
binary variables	50	100	150	200	250	300	350	400	450	500
constraints	1,366	2,731	4,106	5,461	6,826	8,191	9,556	10,921	12,286	13,651
solution time*	305.1	608.0	912.3	1403.9	1785.1	2698.2	3571.2	305.1	607.9	912.3

*The NLP subproblems were solved using CONOPT2 (Drud, 1994) and the MILP subproblems were solved with OSL (IBM, 1991) on a PC Pentium M / 1.6 MHz platform

It can be seen from Fig. 5 that both decomposition strategies showed better performance in comparison with DICOPT++ in terms of computational time. This agrees with the expected behavior that the solution process is improved by solving a set of smaller problems rather than a single large full-scale problem, regardless of the number of scenarios. In Fig. 5*d* it can also be observed that ***Strategy 2*** also shows smaller solution times in comparison with those obtained for ***Strategy 1***. This can also be explained in terms of the size of the subproblems. ***Strategy 1*** solves smaller MINLP dual subproblems, whereas it deals with larger NLP primal subproblems. ***Strategy 2***, on the other hand, solves small MINLP problems in terms of both dual and primal subproblems.

Regarding the quality of the solutions, that is, how different the solution found in the full-scale problem and that in the proposed strategies are, the opposite behavior is observed. This analysis is presented in this work in terms of the objective function in Fig. 6 and it can be observed that ***Strategy 1*** presents slightly better performance in comparison with ***Strategy 2***.

VIII. CONCLUSIONS

This paper showed that in order to solve more realistic problems that encompass multiple time periods it is imperative to rely on decomposition techniques. Two decomposition strategies were proposed and have performed very well in the long-range production planning of a petroleum refinery under uncertainty. ***Strategy* 1** stands for a dual problem that is given by a Lagrangean relaxed problem and whose primal problem corresponds to the original problem with the binary decision variables fixed to the values obtained with the dual problem. The Lagrangean multipliers are updated at each iteration through the subgradient optimization. ***Strategy* 2** differs from ***Strategy* 1** only with respect to the primal problem whose fixed decision variables are not the same. ***Strategy 1*** fixes binary variables, whereas ***Strategy 2*** fixes inventory variables. Both strategies showed to be very efficient for problems that consider uncertainty in the way of discrete scenarios.

Moreover, it is important to note that the paper relied on model instances that could be solved by the decomposition as well as the full-scale methods, since the main objective was to compare the computational effort and the quality of the solutions for all approaches.

Although global optimality is not guaranteed (neither it is in the solution of standard MINLP), such methods perform relatively well and can be extended to large-scale problems.

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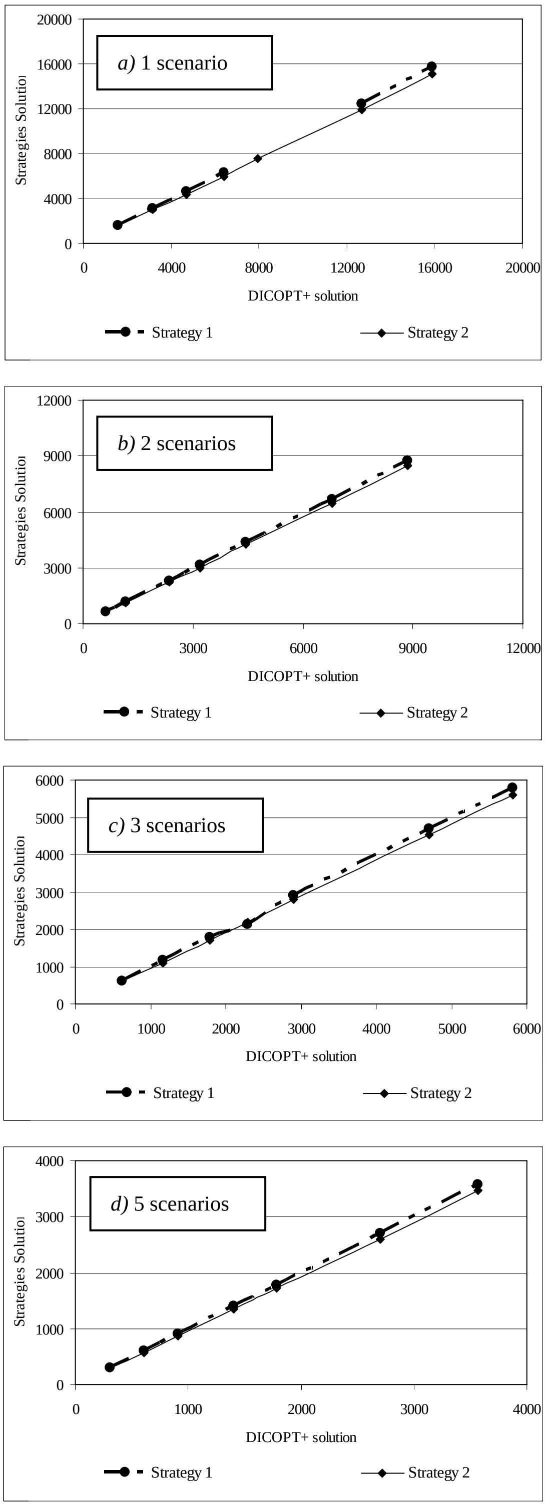

Fig. 6 – Comparison between the objective functions obtained by DICOPT++ and by the proposed strategies.

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