A STUDY OF ULTRASONIC WAVE PROPAGATION IN BONES

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***Abstract*— In the present work the propagation of ultrasonic waves in human bones is modeled by Biot equations introducing in them viscoelasticity so as to take into account attenuation mechanisms; different models for the latter are introduced and compared. Also, Biot equations are numerically solved using two dimensional finite elements. How different porosities affect elastic moduli, phase velocities, viscodynamic and intrinsic attenuation is also studied.**

***Keywords*— Cortical and trabecular bone, Low frequency, Biot theory, Finite elements.**

I. INTRODUCTION

Osseous tissue forms the variety of bones that make up the skeletal system. The matrix is partially organic (collagen fibres in which osteocytes are embedded) and partially inorganic (mineral salts with calcium as the most important mineral). Because of its structure, bone tissue is found in two different types, Compact Bone and Cancellous or Trabecular Bone. Compact bone is very dense and strong, found on the outmost portions of all bones as a protective layer. It contains numerous osteons or Haversian Systems with a central channel through which blood vessels and nerves pass. Surrounding this channel there are multiple concentric layers of tissue known as lamellae. On the other hand, Cancellous Bone is lightweight and spongelike; the interconnected bony pieces forming this characteristic structure are called trabeculae. Trabecular bones have relatively large hollows filled by bone marrow and adipose tissues. Therefore, the relative volume fraction of solid in both structures is different; bone portions with volume fraction of solids below 70% are classified as trabecular, and over 70 % as cortical (Cowin, 1999; Smit *et al.*, 2002; Barkmann *et al.*, 2003; Bossy *et al.*, 2004; Buchanan *et al.*, 2004; Bossy *et al.*, 2005). Biot (1956 a,b,c) established a theory of propagation of elastic waves in a medium composed of a porous elastic solid saturated by a simple and viscous fluid phase. He showed the existence of a shear wave and two compressional waves, a high velocity one (wave of the first kind or Type I wave) corresponding to in-phase motion of solid and fluid and a low velocity one (wave of the second kind or Type II wave) associated to out of phase motion between solid and fluid. These compressional waves had been observed in laboratory experiments with bones, see for example Lakes *et al.* (1986); McKelvie and Palmer (1991); Kacsmarek *et al.* (2002); Lee *et al.* (2002); Cardoso *et al.* (2003).

Numerical simulation of ultrasonic wave propagation in bones using Biot theory has been reported since several years ago by many authors -see Haire and Langton (1999); Hughes *et al.* (2003); Lee *et al.* (2003); Buchanan *et al.* (2004); Fella *et al.* (2004); Wear *et al.* (2005) to name some of them-, but still there are open questions to be answered, as the reader can see below. The aim of this report is to analyze the propagation of ultrasonic waves in human bones and study how different porosities affect elastic moduli, velocities and intrinsic attenuation. Biot theory is used for two dimensional numerical simulation where bones are considered as a biphasic poroviscoelastic material. It is important to remark that the upper bound for the involved frequencies is reached when the corresponding wavelength becomes of the order of the porous size. Experiments involving frequencies beyond this limit must be treated within a different theoretical frame, as stated by the author of the theory (Biot, 1956c).

Following Mellish *et al.* (1989) and Hughes *et al.* (2003), the typical pore radii (mean values and standard deviations) are 0.285 ± 0.050 mm for young normal bone and 0.455 ± 0.130 mm for aged and osteoporotic bone. Bossy *et al.* (2005) reports an average trabecular spacing ranging from 0.5 mm to 2.0 mm; therefore Biot theory cannot be used with bones for frequencies greater than 750 kHz.

Keeping in mind that Biot's theory is applicable at wavelengths much longer than the pore size (corresponding to frequencies within the 0–750 kHz range), and that the Type I and shear waves have a behaviour similar to those in an elastic solid, with high phase velocities, low attenuation and little dispersion, it is necessary to establish the critical frequency f_c that determines whether or not Poiseuille flow occurs. When the frequency range is below f_c, the relative motion of the fluid in the porous is of the Poiseuille type; the coupling between solid and fluid is preeminently viscous, leading to a slow wave of diffusive nature. The assumption of Poiseuille is not established for the higher frequency range $f_c \leq f$; in this regime inertial coupling dominates over the viscous one leading to a truly propagating slow compressional wave. Johnson and Plona (1982) showed that the latter cannot propagate if the viscous skin depth

is less than the porous radius and developed a second frequency limit called the viscous frequency, f_v. The viscous skin depth depends on the frequency and the fluid properties (viscosity and density) as the next section will make clear. Results obtained by Hughes *et al.* (2003) indicate mean values of f_c=4.5 Hz and f_c=705 Hz for normal bone with water or marrow at 20°C, respectively. For osteoporotic bone these same values are now f_c=1.4 Hz and f_c=231 Hz. Therefore, the viscodynamic correction must be introduced in the ultrasonic range, as has been reported in Fella *et al.* (2004). In the present paper, the theory of dynamic permeability and tortuosity presented by Johnson *et al.* (1987) is considered for calculating this frequency correction function.

Other point that is taken into account in the present work is that, when numerically studying how the wave propagation depends on the values of different model parameters, their variation must be done in a consistent fashion. For example, if the bone porosity is diminished, the bulk moduli must be accordingly modified, otherwise phase velocities will be under or overestimated. The Hashin-Shtrikman bounds (Mavko *et al.*, 1998) are used in this work to estimate the dependence of the moduli with porosity.

II. BIOT THEORY

Consider an isotropic, porous solid saturated by a single-phase, compressible and viscous fluid. Let $u^s = \left(u_i^s\right)$ and $\tilde{u}^f = \left(\tilde{u}_i^f\right)$, i =1, · ,d be the averaged displacement vectors of the solid and fluid phases, respectively, where d indicates the Eucliden dimension, i.e., d =2, 3. Also, let

$$u^f = \phi\left(\tilde{u}^f - u^s\right) \tag{1}$$

be the average relative fluid displacement per unit volume of bulk material, where ϕ means the effective porosity. Set u =(u^s,u^f) and recall that

$$\xi = -\nabla . u^f , \tag{2}$$

represents the change in fluid content.

Let $\varepsilon_{ij}(u^s)$ be the strain tensor of the solid. Also, let τ_{ij}, i,j =1, ·· ,d, and p_f denote the stress tensor of the bulk material and the fluid pressure, respectively. Following (Biot, 1962), the stress-strain relations can be written in the form:

$$\begin{aligned} \tau_{ij}(u) &= 2\mu\varepsilon_{ij}(u^s) + \delta_{ij}\left(\lambda_c \nabla . u^s - D\xi\right), \\ p_f(u) &= -D\nabla . u^s + K_{av}\xi. \end{aligned} \tag{3}$$

The coefficient μ is equal to the shear modulus of the bulk material, considered to be equal to the shear modulus of the dry matrix. Also

$$\lambda_c = K_c - \tfrac{2}{3}\mu, \tag{4}$$

with K_c being the bulk modulus of the saturated material. Following Santos *et al.* (1992), Gassmann (1951) the coefficients in (3) can be obtained from the relations

$$\alpha = 1 - \frac{K_M}{K_s}, \quad K_{av} = \left[\frac{\alpha - \phi}{K_s} + \frac{\phi}{K_f}\right]^{-1} \tag{5}$$

$$K_c = K_m + \alpha^2 K_{av}, \quad D = \alpha K_{av}$$

where K_s, K_m and K_f denote the bulk modulus of the solid grains composing the solid matrix, the dry matrix and the saturant fluid, respectively. The coefficient α is known as the effective stress coefficient of the bulk material.

A. The modified moduli for viscoelasticity

It is well established that frequency-dependent propagation velocity and attenuation are the most significative parameters to determine bone quality by ultrasound (Siffert and Kaufman, 2007). Since experimental measurements yield greater attenuation than the predicted by the lossless Biot theory, an attempt is here made to remedy this misbehaviour by introducing attenuation models into the latter. The chosen ones are popular within the geophysics community, and assume that the material is linear, dissipative and causal. The modified moduli K_m, K_s and μ are introduced by application of Biot's correspondence principle (Biot, 1956c; 1962). The poroelastic coefficients of the stress-strain relations are changed by new complex coefficients that depend on the frequency and satisfy the same relations that are satisfied by the elastic moduli. It is proposed to parametrize attenuation in bones through constant quality factor models, since there exist measurements that indicate an almost linear dependence of attenuation with frequency and the quality factor is nearly constant over a given frequency band. The following formulae allow to calculate the viscoelastic moduli for different models. The form of the moduli in the frequency-domain is obtained by Fourier transform of the stress-stain relations. For a detailed description of the differences among the models here presented, see Mavko and Nur (1979); Johnston *et al*. (1979).

B. Liu model

In the linear viscoelastic model proposed by Liu *et al.* (1976) the following formula allows to calculate the viscoelastic moduli:

$$M(\omega) = \frac{M_{re}}{R_M(\omega) - iT_M(\omega)}., \tag{6}$$

Here M represents any of the mentioned moduli and the coefficient M_{re} corresponds to associated elastic modulus (Bourbie *et al.*, 1987). The coefficients $R_M(\omega)$ and $T_M(\omega)$, associated with a continuous distribution of relaxation times, characterize the viscoelastic behaviour of the material and are given by (Liu *et al.*, 1976)

$$R_M(\omega) = 1 - \frac{1}{\pi\hat{Q}_M}\ln\frac{1+\omega^2 T_{1,M}^2}{1+\omega^2 T_{2,M}^2}, \tag{7}$$

$$T_M(\omega) = \frac{2}{\pi\hat{Q}_M}\tan^{-1}\frac{\omega\left(T_{1,M} - T_{2,M}\right)}{1+\omega^2 T_{1,M}T_{2,M}}. \tag{8}$$

Another measurement of the viscoelastic behaviour of the materials is

$$\tan\left(\delta(\omega)\right) = \frac{\Im(M(\omega))}{\Re(M(\omega))} = \frac{1}{Q(\omega)}, \tag{9}$$

being $Q(\omega)$ the so-called quality factor.

$T_{1,M}$ and $T_{2,M}$ are chosen such that the quality factor $Q(\omega)$ is approximately equal to the average value $\hat{Q}_M$ in the range $T_{1,M} \le \omega \le T_{2,M}$. In general, small values of $\hat{Q}_M$

(on the order of ten) correspond to very dissipative materials, and values of $\hat{Q}_M$ on the order of hundred correspond to quasielastic materials.

An advantage of this model over the others here considered is that the complex modulus behaves appropriately within a wide frequency range including the quasiestatic regime, i.e. $\omega \to 0$.

C. White model

This approach introduces a perfect constant Q for all frequencies (White, 1965). It is a simple model where the complex moduli are calculated by

$$M(\omega) = M_{re}\left(1 + i\hat{Q}_M^{-1}\right), \tag{10}$$

D. Zener or standard linear solid model

This model, usually mentioned as SLS, is suitable to represent a distribution of single relaxation peaks centered at each frequency. The viscoelastic moduli are obtained by

$$M(\omega) = M_{re}\frac{1 + i\omega T_{1,M}}{1 + i\omega T_{2,M}}., \tag{11}$$

Also, it is possible consider only a single standard linear solid centered at a given frequency. In this case, the quality factor Q is frequency dependent.

Recall that in all cases the complex moduli are calculated by applying the different models to K_s, K_m and μ.

E. The equations of motion

The equations of motion for a porous medium saturated by a single-phase in the space-frequency domain can be written, following Santos *et al.* (1992), as

$$-\omega^2\rho_b u^s - \omega^2\rho_f u^f - \nabla.\tau(u) = f^s \tag{12}$$

$$-\omega^2\rho_f u^s - \omega^2 g u^f + i\omega b u^f + \nabla.p_f(u) = f^f, \tag{13}$$

where ρ_s and ρ_f are the mass densities for solid and fluid, respectively and f^s and f^f are solid and fluid sources respectively. The total mass density of the solid-fluid or bulk density is

$$\rho_b = (1-\phi)\rho_s + \phi\rho_f. \tag{14}$$

g is the mass coupling coefficient between the solid matrix and the fluid phase. This coefficient represents inertial effects. The parameter b is the viscosity coupling coefficient between both phases. The relationship among b, the fluid viscosity η, and the Darcy's coefficient of permeability k is given by

$$b = \eta/k, \tag{15}$$

while g is related with the tortuosity factor, S as

$$S = \frac{S\rho_f}{\phi}, \qquad S = 1 - \frac{1}{4}\left(1 - \frac{1}{\phi}\right). \tag{16}$$

It was mentioned that the assumption of Poiseuille flow breaks down when the frequencies exceed a value ω_c (Biot, 1956c; Johnson *et al.*, 1987; Carcione, 2001). This value can be accurately estimated according to the formula

$$\omega_c = \frac{2\eta\phi}{a_p^2\rho_f}, \tag{17}$$

where $a_p = 2(5k/\phi)^{1/2}$ is the pore size parameter as function of the permeability k and the porosity ϕ (Johnson, 1982; Bear, 1972; Scheidegger, 1974; Hovem and Ingram, 1979). For frecuencies higher than ω_c, the coupling coefficients depend on the frequency as

$$g(\omega) = \frac{\rho_f}{\phi}\left(S + \frac{F_i(\omega)}{\omega}\frac{\eta}{k}\frac{\phi}{\rho_f}\right), \tag{18}$$

$$b(\omega) = \frac{\eta}{k}F_r(\omega). \tag{19}$$

The function $F(\omega)=F_r(\omega)+ i\, F_i(\omega)$ is a correction factor proposed by Biot (1956c) as an universal function representing effects of frequency for different size of the pores and their geometry. Johnson *et al.* (1987) provided the following modified version that is used in this article,

$$F(\omega) = \left(1 - \frac{4iS^2k}{x\Lambda^2\phi}\right); \text{ where } x = \frac{\eta\phi}{\omega k\rho_b}, \tag{20}$$

and Λ can be calculated from

$$\frac{8Sk}{\phi\Lambda^2} = 1. \tag{21}$$

The parameter Λ has the dimensions of length and it is a geometrical parameter that represent a characteristic of the porous medium.

A second limit called viscous frequency, ω_v, was suggested by Johnson and Plona (1982) as

$$\omega_v = \frac{2\eta\phi}{a_p^2\rho_f\zeta^2} = \frac{\omega_c}{\zeta^2}, \tag{22}$$

being ζ a scaling constant with an empirically derived value of the order of 0.01.

In order to completely define the problem to be studied, boundary conditions must be attached to equations (12-13). Considering from now on a two dimensional medium, let Γ denote the frontier of the domain Ω, with ν being the unit outer normal at the boundary and χ its unit tangent such that $\{\nu,\chi\}$ is an orthonormal system on Γ. The absorbing boundary condition chosen is

$$\begin{aligned} &-\left(\tau(u)\nu.\nu, \tau(u)\nu.\chi, p_f(u)\right)^t\left(u(x,\omega)\right) = \\ &i\omega\mathcal{D}\left(u^s.\nu, u^s.\chi, u^f.\nu\right)^t\left(u(x,\omega)\right), \qquad x \in \Gamma. \end{aligned}, \tag{23}$$

The matrix $\mathcal{D}$ is positive definite and it is given by $\mathcal{D} = \mathcal{Z}^{1/2}\mathcal{N}^{1/2}\mathcal{Z}^{1/2}$, where $\mathcal{N} = \mathcal{Z}^{-1/2}\mathcal{M}^{1/2}\mathcal{Z}^{-1/2}$, $\mathcal{Z} = \begin{pmatrix} \rho_b & \rho_f \\ \rho_f & g \end{pmatrix}$ and

$$\mathcal{M} = \begin{pmatrix} \lambda_c + 2\mu & \alpha K_{av} \\ \alpha K_{av} & K_{av} \end{pmatrix}, \tag{24}$$

F. Phase velocity and attenuation

Plane waves can be used for obtaining the characteristics of waves propagating in a medium. Because the medium is isotropic, without loss of generality the plane waves propagating in the x-direction are

$$\varphi = A_c e^{i\left(\omega t - \vec{k}_c\vec{x}\right)}, \qquad \psi = B_c e^{i\left(\omega t - \vec{k}_c\vec{x}\right)}, \tag{25}$$

where $\vec{k}_c$ is the complex wave vector. Substituting these expressions into equations of motion (12-13) and zeroing the right-hand yields

$$\left(-\omega^2\rho_b + (\lambda_c + 2\mu)k_c^2\right)A_c + \left(-\omega^2\rho_f + Dk_c^2\right)B_c = 0, \tag{26}$$

$$\left(-\omega^2\rho_f - Dk_c^2\right)A_c + \left(-\omega^2 g + i\omega b - K_{av}k_c^2\right)B_c = 0. \quad (27)$$

Taking the determinant of this system of equations equal to zero and considering the complex velocity $v_c=\omega/k_c$, the following dispersion relation is obtained:

$$\left(\rho_b\left(g-\frac{ib}{\omega}\right)-\rho_f^2\right)v_c^4 + \left(\rho_b K_{av} - H_c\left(\frac{ib}{\omega}-g\right)\right)v_c^2 + \left(D^2 - H_c K_{av}\right) = 0, \quad (28)$$

where $H_c = \lambda_c + 2\mu$. The two roots of this second-order equation in v_c^2 correspond to the fast and slow compressional waves. The phase velocity and the attenuation factor can be calculated by

$$v_p = \frac{\omega}{|\Re(k_c)|}, \qquad \alpha = -\omega\Im(k_c), \quad (29)$$

respectively.

G. The effective moduli and compressional velocities

In order to theoretically predict the effective elastic moduli, K_m and $\mu=\mu_m$ of a mixture of a porous solid material and a fluid, generally, it is necessary to specify the volume fractions and the moduli of the constituents. The Hashin-Shtrikman expressions allow to predict the upper and lower bounds of the elastic bulk and shear moduli of the dry matrix without being forced to consider details of the geometries of the phases (Hashin and Shtrikman, 1963; Mavko *et al.*, 1998). A similar approach to calculate upper and lower bounds for other Biot parameters can be found in Norris (1989). For the upper bounds the Hashin-Shtrikman formulae are

$$K_m^+ = K_s + \frac{\phi}{\left(K_f - K_s\right)^{-1} + (1-\phi)\left(K_s + \frac{3}{4}\mu_s\right)^{-1}}, \quad (30)$$

$$\mu_m^+ = \mu_s + \frac{\phi}{\left(\mu_f - \mu_s\right)^{-1} + \frac{2(1-\phi)(K_s + 2\mu_s)}{5\mu_s\left(K_s + \frac{3}{4}\mu_s\right)}}. \quad (31)$$

The lower bounds are calculated by interchanging moduli and volume fractions; the separation between both bounds depends on how different the phases are. Because Biot's effective medium model consider a homogeneous modulus, a possibility is to take the average of the upper and lower bounds, $(K_m^+ + K_m^-)/2$ and $(\mu_m^+ + \mu_m^-)/2$. Notice that there exist other ways of calculating these bounds, e.g. Gassmann's relations, but they are not used here because they are appropriated for quasi-static regimes. Figure 1 shows the separation between the upper and average bounds vs. porosity assuming K_s = 16.74 *GPa*, μ_s =6.25 *GPa* for bone (Smit *et al.*, 2002) and K_{air} =0.01 *GPa*, μ_{air} =0 *GPa* for the air. Therefore, the lower bound for K_m is in the range 0.01 *GPa* to 0.25 *GPa* and for μ_m is zero. The fast and slow compressional velocities can be estimated from the above results. Several considerations concerning the pore size, the porosity, the permeability, the tortuosity and some geometric details in addition to previous elastic moduli have been taken into account to plot velocities vs. porosity in Fig. 2. This figure illustrates the behavior of elastic and anelastic velocities for different viscoelastic models with the quality factor $\hat{Q}_M = 30$ if the bone porous matrix is saturated by water. Note that both type I and type II wave velocities tend to the speed of sound in water (1563 ± 25 m/s), Nicholson *et al.* (2000) as the porosity increases (lower solid fraction). All Biot viscoelastic velocities are greater than elastic ones and the major difference is about 10 % for the Liu model. The behaviour of the attenuation coefficient as a function of the porosity is shown in Fig. 3. While the attenuation of the fast wave increases slowly when the solid fraction decreases, the attenuation of the slow wave presents high values for cortical bone porosities. Figure 4 represents the attenuation coefficient as a function of the frequency for trabecular and cortical structures with ϕ=0.8 and ϕ=0.05, respectively. The curves correspond to $\hat{Q}_M = 30$, $T_{1,M}$ =1 *s* and $T_{2,M}$ =1.10^{-12} *s*. It must be noticed that in these models the type of dispersion is anomalous, i.e., the phase velocity increases with frequency. In addition, it is possible to analyze how the attenuation of fast and slow waves depends on $\hat{Q}_M$. Considering a cancellous bone, Fig. 5 displays curves of attenuation vs. frequency for $\hat{Q}_M = 30$ and $\hat{Q}_M = 10$ (Liu model). The attenuation is notably stronger for the fast wave; and for this model these results are close to the experimental results of Serpe and Rho (1996).

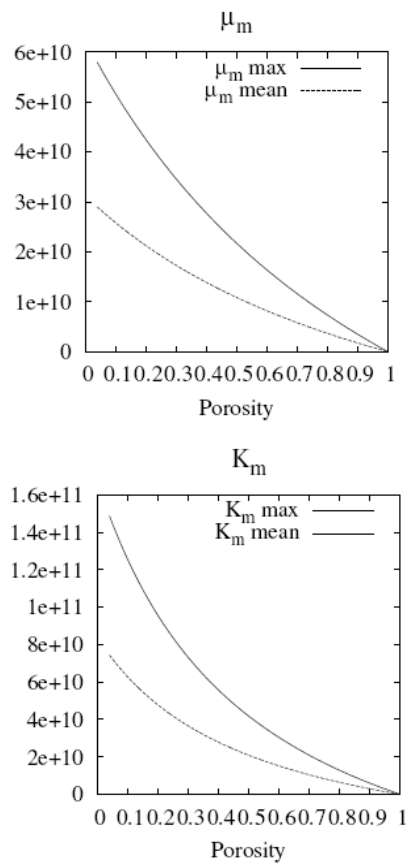

Figure 1: Effective moduli (measured in Pa) of the porous matrix of dry bone. Bottom: Bulk modulus. Top: Shear modulus.

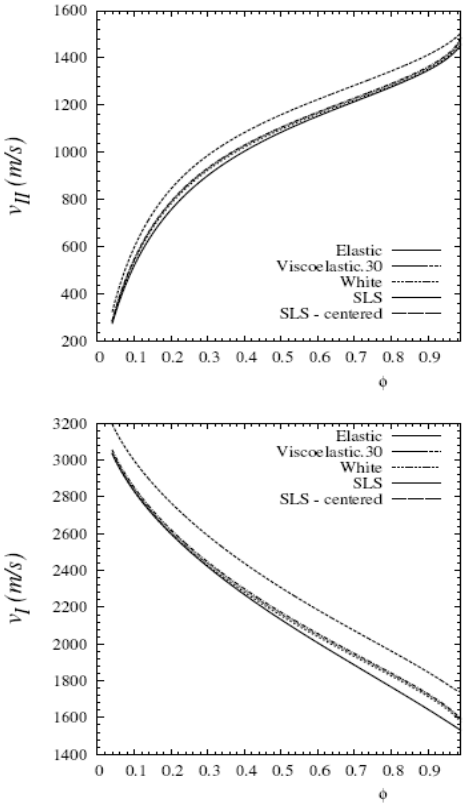

Figure 2: Compressional Biot velocities as a function of the porosity for different attenuation models. The single SLS is centered at f0 = 300kHz. Bottom: Type I. Top: Type II.

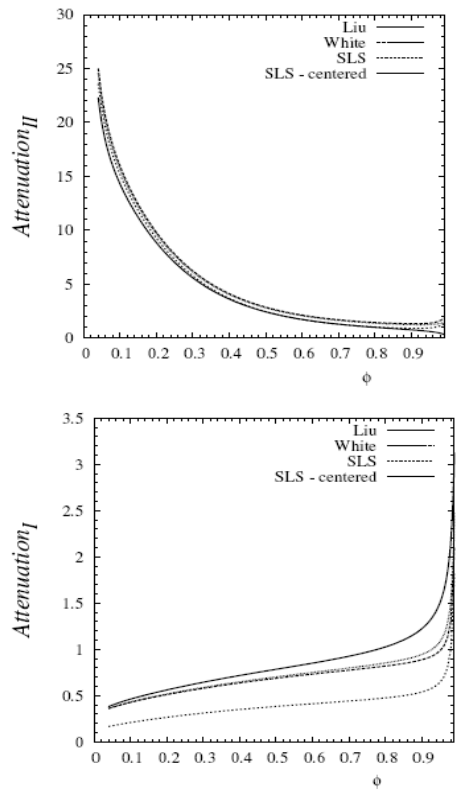

Figure 3: Attenuation coefficients, measured in dB/cm, as a function of the porosity and different attenuation models for Type I wave (bottom) and Type II wave (top). Again, the single SLS is centered at f0 = 300 kHz.

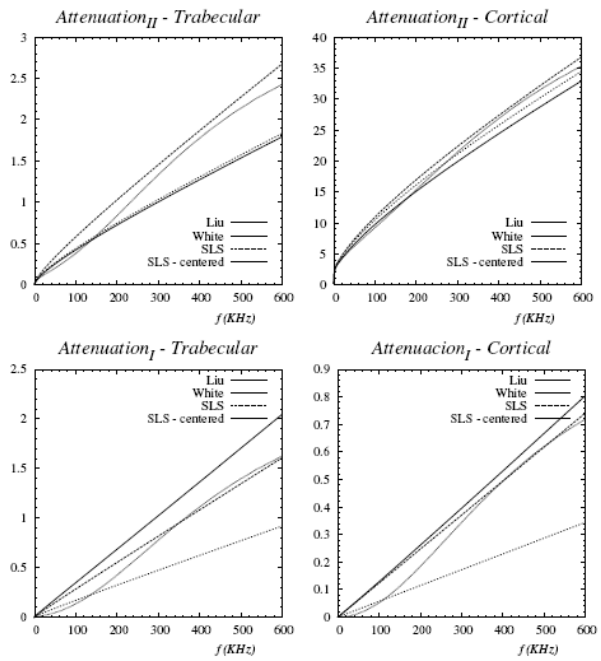

Figure 4: Attenuation coefficients, measured in dB/cm, for Type I and Type II waves in trabecular and cortical bones as a function of the frequency, for different attenuation models. As in previous figures, the single SLS is centered at f0 = 300 kHz.

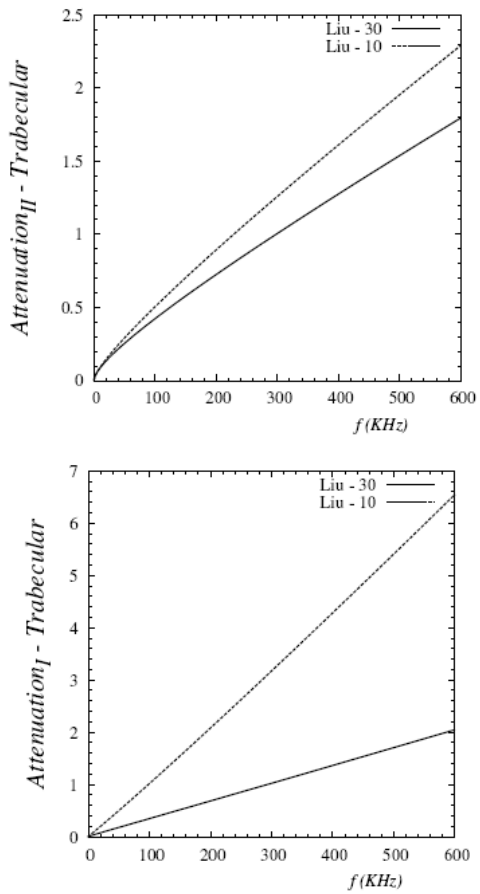

Figure 5: Attenuation coefficients (dB/cm) as a function of the frequency for Type I wave (bottom) and Type II wave (top). Liu model with $\hat{Q}_M = 30$ and $\hat{Q}_M = 10$.

Table 1: Estimated parameters for cortical and trabecular bone. CB: Cortical bone, NTB: Normal trabecular bone, OTB: Osteoporotic trabecular bone.

	CB	NTB	OTB
ϕ	0.05	0.80	0.90
K_m [GPa]	7.23	0.642	0.298
μ_m [GPa]	2.84	0.368	0.175
ρ_m [g cm^{-3}]	1.96	1.96	1.96
κ [cm^2]	4 10^{-9}	5 10^{-4}	1 10^{-4}

III. NUMERICAL APPROXIMATION

Naturally, the space–frequency domain is chosen to describe phenomena of dissipation and anelastic attenuation suffered by the different type of waves here considered. Moreover, this domain allows to compute the solution at the current time without the knowledge of the time history of the system, that is, the solution is computed for each temporal frequency and a finite number of them are required to obtain the solution in the space–time domain via inverse Fourier transform (Douglas *et al.*, 1994; 1993; Gauzellino *et al.*, 2001).

As already mentioned, the numerical resolution of Eq. (12-13-23) is performed using piecewise linear finite elements.

A. Wave propagation modeling

A numerical experiment to analyze the influence of porosity variations in the observed traces within a piece of bone is designed. The following simple source is chosen:

$$f(x,t) = -2\zeta(t-t_0)e^{-\xi(t-t_0)^2}\delta(x-x_s) \qquad t \geq 0, \qquad (32)$$

being f_0 = 300kHz the principal frequency of the source, with $\xi = 8 f_0$ and $t_0 = 1.25/f_0$. A finite number of frequencies between 0 kHz and 600 kHz is used in order to approximate $u^s(x, t)$ and $u^f(x, t)$. Table 1 shows estimated parameters at different porosities to be used in the simulation of wave propagation. The fluid phase is water and its parameters $K = 2.3GPa$, $\rho = 1gcm^{-3}$ and $\eta = 0.01Poise$ are taken from (McKelvie and Palmer, 1991; Buchanan *et al.*, 2004). The bone specimen is represented by a square sample with sides of 30 mm. The signal transmitted through the bone is recorded by a receiver located at 30 mm from the source. Figure 6 illustrates the signals acquired, $u^s(1-\phi) + u^f \phi$ for different porosities. It can be seen that the slow wave amplitude is relatively important in cancellous structures contrarywise to what happens in a compact structure. In trabecular bone, the velocities of the fast and slow waves are similar due to the relatively high percentage of fluid. Therefore, when the porosity is very high, what is recorded is a superposition of the two waves. It has been already established that osteoporotic bone can be also characterized by low density. For the fast wave, if the density decreases to 20 %, the velocity increases about 8 % and the amplitude increases too; on the contrary, for the slow wave the changes are not significant.

IV. CONCLUSIONS

It is long established that the Biot theory for wave propagation in a fluid saturated porous medium may be a useful tool to model ultrasound probing of both trabecular and cortical bone. In this paper an upper bound for the frequency spectrum of the acoustic source has been introduced; this upper bound is far lower than frequencies used in the literature assuming the validity of Biot theory.

An open question is which underlying loss process within the bone leads to wave attenuation. In the present effort this mechanism is accounted for assuming viscoelastic behaviour, modeled by several different mechanisms. The propagation of fast and slow waves travelling through both cancellous and cortical structures is fairly well followed by all implemented possibilites.

Is is also observed that the measured signal, when the target is trabecular bone is an overlap of both slow and fast waves, rendering meaningless the idea of extracting information by analyzing just one of the two involved propagation modes.

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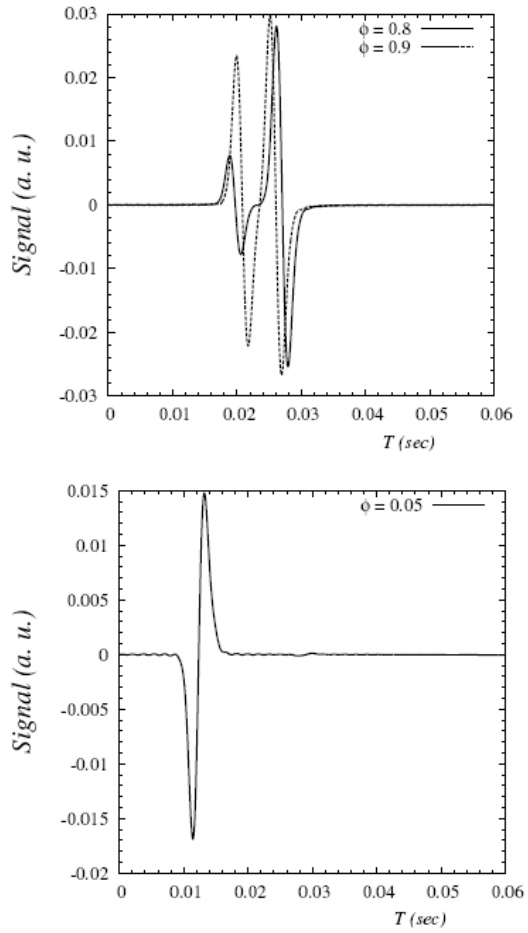

Figure 6: Signals acquired, $u^s(1-\phi) + u^f \phi$, for samples with different porosities; the bottom one corresponds to a cortical bone, the top one to different trabecular bones.

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