FINITE ELEMENT MODELING OF FOOD COOKING

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Abstract - In the last few years there has been an evident rise in the production of previously cooked, ready-to-serve meals.

At a global level, the studies on this matter have yet to develop. Heat (and mass) transfer are modeled to predict thermal histories, microorganisms destruction and, finally, the cooking time. Both the experimental and modeling problems are quite complex, due to an important contribution of the radiation in the oven to the heat transfer and the simultaneous water loss by means of evaporation, porosity and volume variation, crust formation, etc. Nowadays the modeling of the process is carried out using finite differences or finite elements, contemplating important simplifications.

To simulate the cooking of different foods in a convection oven, a commercial finite elements software (ALGOR) is used.

The three-dimensional mesh of finite elements is formed using “Brick” elements, proper of the ALGOR software heat transfer module. The thermal variable properties of the food are considered, including the specific apparent heat, which involves the evaporation of water in the superficial crust. The convective and radiation components of the heat transfer in the air-food system are also taken into account. This is done with heat transfer coefficients determined in specific experiments for the utilized oven.

The obtained results are validated against experimental data, from cooking runs of the same foods in the determined oven.

Keywords – Cooking, Finite element, Prepared meals.

I. INTRODUCTION

Nowadays, cooking is a relevant technological process in different food industries. It is especially important in the fast food sector, catering services and production of prepared meals (ready-to-serve). From a safety point of view, cooking may ensure the elimination of harmful pathogens (principally Escherichia coli O157:H7).

The rise on the production of pre-cooked meals throughout the whole world in recent years has been more than noticeable. These products are baked in continuous or batch equipments and are commercialized frozen or refrigerated, ready to be reheated and consumed.

One fundamental condition for the cooking stage is that it must be done with as little thermal abuse as possible. For that, a special emphasis in the study of the heat transfer is necessary in this process, to find time-temperature relations that ensure the completion of the process from the microbiological point of view without losing any of the nutritional or organoleptic characteristics of the food.

Being able to simulate the heat transfer during the cooking process is essential for the specification of the cooking conditions (cooking time, temperature and air circulation) and the study of quality factors (crusting, dehydration, general appearance).

The modeling of the cooking process is, nowadays, done by means of finite differences (FDM) or finite element models (FEM), assuming important simplifications (Nicolaï et al., 1995). Most of the published models refer to regular shapes and meat products: meatballs (Hung and Mittal, 1995), meat loaves (Holtz and Skjölderbrand, 1986), deep-fat frying of chicken drum (Ngadi et al., 1997), hamburger patties (Ikediala et al., 1996; Zorrilla and Singh, 2000), both of them without shrinkage; chicken patties (Chen et al., 1999), patties with radial shrinkage (Zorrilla and Singh, 2003), meat cooking (Salvadori and Mascheroni, 2003; Purlis and Salvadori, 2005).

Another aspect to consider is that the simulation of heat transfer in cooked food trays presents two conditions that make it considerably more difficult: the fact that food is heterogeneous and generally anisotropic, and the lack of information and complexity of determination of the physical properties that are involved (Purlis and Salvadori, 2005).

In this sense, the finite elements method is more flexible than that of finite differences, for it permits the modeling of multidimensional irregular geometries, considering volume deformation and heterogeneous, non-isotropic foods.

The objectives of the present work are:
- To study the feasibility of using a finite elements commercial software to model the cooking of different prepared foods: potato and meat pie, beef pieces and lasagna.
- To systematize the use of the prediction software by establishing standardized procedures for the
development of the finite elements mesh, the expressions for the boundary conditions and the calculation of the variable physical properties.

- To verify the numerical results with experimental ones, obtained in an electric oven with controlled temperature, with natural or forced convection.

II. METHODS

A. ALGOR

The modeling and simulation of the cooking of the different foods was performed using ALGOR v. 13.36, (ALGOR, 2003) commercial finite elements software.

This software is basically oriented to the simulation of mechanical events. Nevertheless, it is equipped with additional modules that are developed to solve both stationary and transient heat transfer problems. It also allows the incorporation of variable properties, an essential feature when simulating the proposed model systems.

A few consecutive stages constitute the modeling and simulation process:

- Preprocessing
- Processing
- Post processing

The environment for ALGOR is the program FEMPRO, in which the kind of analysis to be performed is set at the beginning of the simulation.

During the Preprocessing stage, the program Superdraw is used, since it provides the necessary tools to build the geometry of the model (with the menus Add, Construct y Modify) and then generate the mesh to obtain the finite elements (menu Fea Mesh).

Both two-dimensional and three-dimensional models can be generated (2-D, Brick, Tetrahedral, and Rod Plate). The mesh is made either manually or automatically; according to different esteric options (between nodes and between objects).

The user is able to confer the geometric zones with different physical properties or kinds of elements by defining Groups.

In order to assign the boundary conditions, the Surfaces that contain the elements of the system that are in direct contact with the external medium must be defined.

Layers are useful when working with complex models, as they can “hide” certain parts of the drawing (if they are combined with visualization tools). This improves the perspective of the model during the construction and optimization of the latter.

The steps following the preprocessing stage, such as the definition of the working conditions, the depuration of mistakes, the definition of material and thermal properties, are carried out with FEMPRO, except for the numbering of the nodes (which is done using Superview). Also, Superview can detect construction mistakes in the finite elements model and allows the definition of the data to be included in the final results report.

The Processing stage is initiated in the FEMPRO environment. In this analysis, the transient heat transfer mode solves the following microscopic balance:

\[ \rho C_p \frac{dT}{dt} = k \nabla^2 T \]  

where \( T \) is the temperature, \( \rho \) is the density, \( C_p \) is the specific heat and \( k \) the thermal conductivity.

The initial condition could be a uniform temperature \( T_i \) (potato and meat pie and lasagna simulation) or the temperature profile of a previous stage of simulation (beef pieces).

The boundary condition could consider convection, radiation and a prescript flux Q (surface evaporation).

\[ k \nabla T = h(T_{oven} - T_s) + e \sigma (T_{oven}^4 - T_s^4) + Q \]  

where \( h \) is the heat transfer coefficient, \( T_{oven} \) is the oven temperature, \( T_s \) is the surface temperature, \( e \) is the emissivity, \( \sigma \) is the Stephan-Boltzmann constant.

Basically, the simulation stage consists of the definition of the parameters that enable us to establish, among others, the number of steps, the time step, the kind of boundary condition and the load curves to consider variable \( T_{oven} \) (if necessary). These parameters also enable us to link the simulations with previous runs, specify nodes whose results will be printed and specify the number of steps between each reformulation of the properties mesh. Once these parameters are assigned, the proper calculation stage begins.

The last stage (Postprocessing) is completed in the Superview environment, which shows on the three-dimensional model the temperature or heat transfer flux values, using a predetermined color scale. Another tool from FEMPRO, Monitor, displays the temperature vs. time curves for the specified nodes. Besides, it permits the generation of a video file (.avi) that shows the progress of the simulation in time in transient problems.

Physical properties

The software is provided with a library of physical properties for different materials (metals, cement, etc.), which does not include food products. However, physical properties libraries for other isotropic or orthotropic (that includes those dependant on temperature) materials can be generated with the application Matlibs.

In particular, the “Foods” library, which contains information on the physical properties of different products as a function of the temperature, was generated with the software during previous cases.

In this work, values of the properties corresponding to the groups that form the different study systems (mashed potato, minced meat, spinach, lasagna pasta, tomato sauce and beef) were added to the previously mentioned “Foods” library. These values were calculated according to their composition and temperature, using the equations proposed by Choi and Okos (1986) and incorporated to the library as a table (Fig. 1).
B. Finite Elements Food Models

The cooking of three different food models was simulated: a potato and meat pie ("pastel de papas"), beef pieces of semitendinous muscle ("peceto") and lasagna with vegetable filling.

In the following paragraphs, a complete description of each system is included.

Potato and meat pie

This meal consisted of a 0.01m thick layer of minced meat at the bottom and a 0.01m thick layer of mashed potatoes on top, contained in an individual aluminum tray (of 0.14 x 0.10 x 0.045m).

To simulate the cooking of this meal, the following model was developed: regular mesh (Fea Mesh, 8 point), three-dimensional, with 1910 Brick elements with trapezoidal section. Due to the symmetry of the system, only one forth of the system is employed.

The FEA model considers three different groups: mashed potatoes, minced meat and aluminum. In the foods, the dependence of the physical properties on the temperature was considered.

The boundary condition was only convective on the surface of the pie, with \( h = 20 \text{ W/(m}^2\text{C)} \) (the heat transfer coefficient \( h \) was measured in each cooking experiment, more details in Section C). Besides, on the tray surfaces (lateral and bottom) a radiant condition was contemplated, with \( e=0.1 \) (average emissivity of polished aluminum, Wen and Mudawar, 2005).

The simulation of the heating process was carried out with an initial temperature of 15°C for the product and 210°C for the ambient, and a time step of 30 seconds.

The resultant FEA mesh of the aluminum tray and the slice of the pie are shown in Figs. 2 and 3, respectively. In this figures the different surfaces or groups that were considered can be easily visualized, since ALGOR identifies each one of them with a different color.

Beef pieces

Beef pieces ("Peceto") of trapezoidal section and an approximate weight of 170 g were considered to simulate its cooking.

Two different FEA models were developed and tested:

BP1.- Regular mesh (Fea Mesh, 8 point), three-dimensional, resulting in 2500 Brick elements of trapezoidal section.

BP2.- Irregular mesh, automatically generated in two dimensions and then extruded to construct the three-dimensional model, with 850 elements.

Variable thermal properties of beef and convective boundary condition were used (the heat transfer coefficient \( h \) was measured in each cooking experiment, more details in Section C). Also, variable oven temperature was considered (the experimental curve was acquired and incorporated to the simulation through a load curve).

Figures 4 and 5 show the xy plane (2D) and the isometric (3D) views of the finite elements mesh of the model BP1. Accordingly, Figs. 6 and 7 shows the same views of the model BP2.
Because of a restriction of the software, inner generation of energy depending on time cannot be contemplated. Therefore, to take into account the superficial evaporation and consequent dehydration a negative heat flow term was included on the sides resulting of the intersection of two faces of the sample.

To consider the shrinking of the sample during the cooking process, parallel schemes of the modeling system (with identical meshes) were built, with a determined size reduction (three consecutive reductions from the initial size to the final size, observed in each sample after cooking). Such models were executed correlatively.

Lasagna
In this particular case the characteristics of the model used were the following: Regular mesh (Fea Mesh, 8 point), three-dimensional, with 3100 Brick elements of trapezoidal section. Due to the symmetry of the system, only one forth of the latter was considered for the simulation.

The model was constituted by four groups; three of them were foods and one aluminum. Figure 8 represents the distribution of these groups.

For all the faces of the forth of tray that were exposed to the ambient, a convective boundary condition was considered, with heat transfer coefficients of 20 Wm\(^{-2}\)C\(^{-1}\) and 40 Wm\(^{-2}\)C\(^{-1}\) for natural and forced convection respectively (the heat transfer coefficient h was measured in each cooking experiment, more details in Section C).

The initial product temperature was 15°C and the ambient temperature, constant, was 210°C. The time step was 5 seconds.

The finite elements mesh for the aluminum tray was the same one that was used for the simulation of the pie cooking (Fig. 2).

Figure 9 shows the finite elements mesh for this food in particular.
In the simulation of the heating of lasagna, the superficial evaporation was contemplated by including in the material library an apparent specific heat $c_{eq,j}$ (Eq. 3), which includes the sensitive heat and the latent heat:

$$c_{eq,j} = c_j + \Delta H_{\text{evap}} x_{w,j} / \Delta T$$  \hspace{1cm} (3)

where $c_j$ is the specific heat of each component (spinach, pasta and sauce), $\Delta H_{\text{evap}}$ is the latent heat of the water evaporation (=2257kJ kg$^{-1}$), $x_{w,j}$ is the water content of j component (mass fraction) and $\Delta T = 5^\circ C$.

### C. Cooking experiments

In order to validate the obtained results for the different foods through the numeric method, cooking experiences in an electric oven (Ariston FM87) were performed. The oven has seven different cooking modes (with or without ventilation, with or without grill, etc.) and very accurate control of the inner temperature.

Individual aluminum trays (of 0.14 x 0.10 x 0.045m) were used to contain the two prepared meals simulated in this work, and the tray was placed over the oven tray. The meat pieces were cooked placing them in the oven tray.

Each of the three experimental systems has the characteristics of the simulated ones:

- **Potato and Meat Pie**: consist in a 0.01m thick layer of minced meat at the bottom and a 0.01m thick layer of mashed potatoes on top.

- **Lasagna**: consisted of two 0.001m thick layers of pasta and one 0.09m thick layer of spinach filling between the other two. The lasagna was covered with a 0.06 thick layer of tomato sauce (Fig. 8).

- **Beef pieces**: Two pieces of trapezoidal section, of 171 g and 174.7 g respectively, were cooked in the oven (Fig. 10). The experimental volume shrinkage of both samples was 16% approximately. The first sample was processed in natural convection condition ($h=16$ Wm$^{-2}$C$^{-1}$) and the other in forced convection mode ($h=40$ W/m$^2$C$^{-1}$).

As it is shown in Figs. 8 and 10, T-thermocouples were placed in the inside of the products to measure its temperature during cooking. An additional thermo-couple was used to measure the temperature of the air close to the sample.

The cooking was considered completed only once the thermal center of the samples had reached 72°C, in accordance to the adopted microbiological criteria (FDA, 1997).

The heat transfer coefficient ($h$) used in the simulation were measured in each experimental run using a heat flow sensor (Omega HFS23) which is attached to the surface of the food product.

### D. Results and Discussion

**Potato and meat pie**

As it was mentioned, this software allows the visualization on the three-dimensional model of the temperature and heat flow results, in a predetermined color scale. Figure 11 shows a screen of Superview that displays the temperature results of the Potato and meat pie model, for an intermediate time step.

The graph in Fig. 12 presents the thermal history of three locations in the pie model: the thermal centre, an intermediate point situated on the diagonal and a point that is adjacent to the base.

The experimental temperatures were graphed in a continuous curve and the numerically calculated values with symbols.
The average relative error between the experimental and numerical temperatures, for all three of the locations TC1, TC2 and TC3, was of 15% with a standard deviation of 6.3%. The predicted cooking time, defined as the time to reach 72°C in the thermal centre, was 1020 seconds and the experimental one was 860 seconds.

**Beef pieces**

The dependence of the solution on the kind of mesh and the time step (1 and 5 seconds) was analyzed.

The thermal histories from the cooking of the model system “peceto” in the oven with natural and forced convection conditions are shown in Figs. 13 and 14 respectively. The accuracy of the predicted results is similar for both meshes and time steps.

**Lasagna**

Figures 15 and 16 present the thermal histories, both measured and calculated for three different positions in the studied system, in the oven with natural and forced convection, respectively. The three locations were: the thermal center (TC2) and two intermediate points between the center and the surface (TC1 and TC3). The experimental temperatures were graphed in a continuous curve and the numerically calculated values with symbols.

The results displayed in these figures state that the proposed numerical model predicts accurately the experimental behavior of the system: the average relative error between experimental and calculated values for the three positions TC1, TC2 and TC3, is 6.7% and 10.6% for natural and forced convection cases respectively.

**III. CONCLUSIONS**

The results obtained with ALGOR indicate that this software is a useful tool to study the cooking of food products such as the three systems considered in this work: potato and meat pie, beef pieces and lasagna. All of them present complex characteristics (anisotropics, multicomponents and with variable physical properties).

Although some difficulties were found trying to consider directly the superficial evaporation, due to specific limitations of the software (it does not permit the inclusion of internal sources of energy generation in...
function of the nodal temperature), the incorporation of a specific effective heat was assertive enough to counterweight the problem.

The construction of parallel schemes (with identical meshes) of the model system to contemplate the shrinkage of the sample during cooking was effective. The obtained numerical results proved to be very accurate when they were compared with the experimental temperature profiles.

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