MODELLING AND SIMULATION OF THE HEAT TRANSFER ALONG A COLD ROLLING SYSTEM

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Abstract— **In the industry of flat rolled sheet product, strip crown and shape has been one of the most important factors for quality assurance, productivity improvement, cost effectiveness, and customer satisfaction. The transient work roll profile influences load distribution and imprints an undesirable profile on rolled strip. Precise prediction of the work roll thermal profile facilitates adjustment of crown control devices for crown compensation and shape correction.**

Many studies, including numerical methods, statistical equations, and analytical solutions are proposed in the literature. This article proposes a semi-analytical solution for the work roll subjected to predict transient thermal profiles of work rolls with multiple cooling / heating zones. It was derived from the heat balance equation using the finite difference method and Runge-Kutta method that are two semi recursive analytical solutions, developed to update, the work roll temperature distribution within a very short computing time. The model suggested is used for the numerical simulations in rolling. In this paper, presenting the equations describing a system part is interesting. An evaluation of this work is made through discussing the results and finally, some prospects are evoked

Keywords— **Modelling, cold rolling, thermal profile, work roll, heat exchange.**

I. INTRODUCTION

In rolling process, large amount of heat generated in the roll bite transfers to work rolls and strip (Rabbah and Bensassi, 2007b). Work rolls are cooled by cooling medium in both entry and exit sides of the mill. Transient cooling behavior of the roll affects temperature distribution and thermal profile.

The strip deformation is caused by several factors; of which we quote the irregular distribution of the liquid rate flow along the rolls. A direct consequence of this phenomenon is the non-homogeneous temperature profile, which causes a dilation of the work rolls in various points (Rabbah and Bensassi, 2007a). One of the effects of this dilation is the strip shape disturbance at the exit of the roller (Rabbah and Bensassi, 2006). To remedy this problem, we are compelled to study the rolls behavior affected by the temperature distribution and thermal profile.

The shape quality of the cold–rolled strips is very important. Temperature rolling is also used to improve the surface quality of the strip and the flatness properties. To optimize these final properties of cold–rolled steel, the elongation of each strip product must be strictly controlled. Therefore, it is particularly necessary to control the cooling headers system, and thereafter, to realize the desired thermal profile.

Many studies have been published to predict thermal profiles based on finite difference methods (Ginzburg, 1993; Poplawski and Seccombe, 1979; Yarita *et al.*, 1979; Kawanami *et al.*, 1985; Tooru *et al.*, 1975; Bennon, 1985) and even simple closed-form formulations (Yasuda *et al.*, 1987). Pallone has, for the first time, derived equations of temperature distributions in work rolls with two cooling/heating zones using Laplace transform and Cauchy's residue theory (Pallone, 1983). Even if the model was later verified by Somers *et al.* (1984); the usage of the model, however, should be limited only for the same width continuous rolling since only two heat transfer zones were considered in the derivation.

The model was later expanded by Guo (1993b) to accept the time discrete multiple zone cooling/heating boundary condition. This modification was adopted for various strip widths and cooling control zones and making it possible for the model to be used in the on-line control. This expanded model showed some disadvantages such as the required large computing time and memory storage. To cope with insufficiency, Guo (1993a) further converted these equations into recursive equations and introduced a hybrid method for fast computation and small memory storage. The model was verified to obtain an empirical heat equation using Powell's method and three measured work roll profiles from a production hot strip mill (Guo *et al.*, 1993).

This article proposes a semi-analytical solution to solve temperature field of the work roll which is subjected to various cooling and heating boundary conditions. The model is particularly adapted to the cold rolling system operation in very low thickness, in order to predict the shape of a strip. The governing basic equations and the boundary conditions associated will also be presented. The various methods of the resolution of the heat equation are quoted in the fourth part. The article ends in an interpretation with an evaluation of the results.

II. THERMAL PROFILE DISTRIBUTION

Several effects act on the strip and work rolls shape. For example, we find the thermal profile that is directly due to the irregularly distributed temperature, the strip width, the separation force, the lubrication properties, the friction and the mill reaction between the rolls and the strip (Guo, 1993b; Guo *et al.*, 1993).

As shown in Fig. 1, there are four heat transfer regions at each cooling/heating zone along the axial direction (Guo and Malik, 2001; Guo, 1994): the roll bite (I), the exit side (II), the roll contact (III), and the entry side (IV). The work rolls are subjected to rolling heat at region I –part of contact with the strip- without any source outside the contact port. The short contact of the work roll with the strip increases its temperature which decreases as soon as the contact finishes, that involves its deformation (Picqué, 2004). Both region II and IV – depending on rolling practice – are cooled by air, emulsion or by water. Region III is normally a cooling zone since the back-up roll temperature is lower than the work roll.

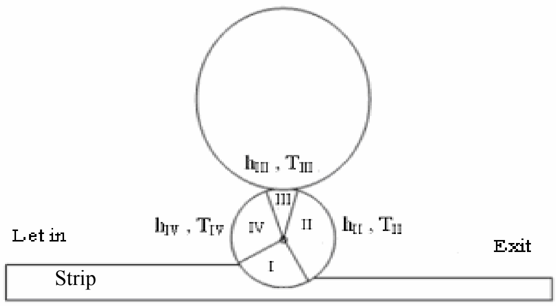

Fig. 1. The 4 borders of the work roll along its circumference.

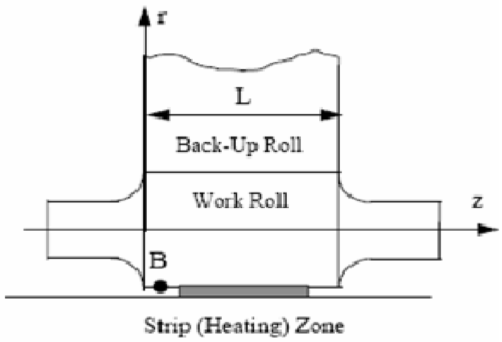

Fig. 2. The work roll direction.

The whole model consists of two elements, predicting at the beginning of rolling the temperature distribution within the work roll and then converting this to the work roll radial expansion. The heat transfer model is given by:

$$\frac{\partial T}{\partial t} = \alpha.\Delta T + \frac{\theta_I}{\rho C \pi r} q - \frac{\theta_{II}}{\rho C \pi r} h_{II}(T - T_{II}) - h_{IV}(T - T_{IV}) \quad (1)$$

with ΔT is the Laplacien of T.

The heat exchange between the strip and the 4 zones of the work roll is carried out gradually zone I to zone IV, passing by two areas II and III. Since area IV is more cooled compared to the others, the exchange between the first and the fourth zone are more dominating in the cold rolling operation (Fig. 2). Moreover, Patula (1981) showed that the great temperature gradient in radial and circumferential directions is restricted in a very thin layer of the work rolls surface. However, we consider only axial conduction (Dhir, 2002; Jarrett and Allwood, 1999). By neglecting the radial and circumferential conditions (Guo, 1993a; Guo, 1994) Eq. (1) can be written as follows:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2} + \frac{2}{\rho c \pi d} Q - \frac{2(\theta_{II} + \theta_{III} + \theta_{IV})}{\rho c \pi d} h_{IV}(T - T_{IV}) \quad (2)$$

with :

$$\begin{aligned} Q = \theta_I q - \theta_{II} h_{II}(T - T_{II}) - \theta_{III} h_{III}(T - T_{III}) \\ - \theta_{II} h_{IV}(T - T_{IV}) - \theta_{III} h_{IV}(T - T_{IV}) \end{aligned} \quad (3)$$

The axial length of the roll is divided into M zones equal width (z_1, z_2, ..., z_i, ...,z_M) (Fig. 3). The temperature of the i^{ème} zone is T_i, with the contribution of the four areas heat previously quoted in figure 1.

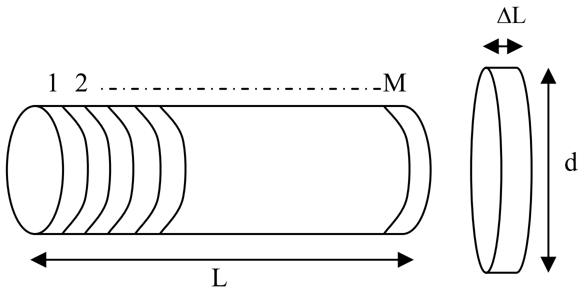

Fig. 3. Axial cut of the cylinder in M set of circular disk.

The temperature differential equation T(z,t) for the ith section, can be written as follows:

$$\frac{\partial T_i}{\partial t} = \alpha \frac{\partial^2 T_i}{\partial z^2} + \frac{2}{\rho C \pi d} Q_i - \frac{4}{\rho C d} h_{IV}(T_i - T_{IV}) \quad (4)$$

with: $i = 1, 2, \ldots, M$.

The Boundary conditions associated can be obtained by the continuity of the temperature conditions and those of the interface heat and to rolls sides:

$$\begin{aligned} &T_i(z_i,t) = T_{i+1}(z_i,t) \qquad \frac{\partial T_i(z_i,t)}{\partial z} = \frac{\partial T_{i+1}(z_i,t)}{\partial z} \\ &T_M(z_M,t) = T_{IV} \qquad T_1(z_0,t) = T_{IV} \end{aligned} \quad (5)$$

and the initial condition:

$$T_i(z,0) = T_0. \quad (6)$$

According to the basic assumptions quoted in (Guo, 1993b; Guo, 1998), the heat flow *Q* can be treated as constant in a small time interval. The roll thermal expansion results from the irregular axial heating, whereas, cooled surfaces are due to the change of the strip width on one hand and to roll zones control by the cooling system on other hand. Table 1 contains the operating condition used in this analysis.

In the following section we will first introduce the various methods to solve the thermal profiles of the work rolls.

Table 1: Parameters of the operating conditions.

Parameters	Numerical values	Units
α	0.288	$W.m^{-1}.K^{-1}$
ρ	1310	$Kg.m^{-3}$
C_p	557.7	
c	Cp* ρ=730587	$J.kg^{-1}.k^{-1}$
d	0.45	m
L (Length)	1.35	m
Q_i	2	$W.m^{-2}$
T_{IV}	313	°K
T_0	223	°K
h_{IV}	10	$W.m^{-2}.K^{-1}$

III. NUMERICAL RESOLUTION

The method of lines (MOL) involves two steps. The first step is the spatial discretization, in which the spatial variables are discretized by means of finite difference methods. After the spatial discretization, a system of ordinary differential equations (ODEs) is generated. The second step is to integrate this ODE system by some standard method to solve the ODE. The method of Runge-Kutta is generally most used (Sadiku and Obiozor, 2000).

A. Finite difference method

In this method, we started by the discretization of Eq. (4) by applying the finite difference method (FDM). This method is based on a recursive equation that counts on a previous solution. It substantially reduces the computing time. We replace the instantaneous temperature expression $T(z,t)$ by that in $(n+1)$ points of spatial discretization corresponding at $(n+1)$ temporal intervals, so that:

$$\begin{cases} z_i = i\,\Delta z \\ t_k = k\,\Delta t \end{cases} \Rightarrow T(z_i, t_k) = T_i^k$$

With Δz and Δt are the space and temporal sampling step respectively.

- The temporal derivative expression:

$$T(z_i, t_k + \Delta t) = T(z_i, t_k) + \Delta t \left.\frac{\partial T}{\partial t}\right|_{z_i}^{t_k} + \vartheta(\Delta t^2)$$

Either:

$$\left.\frac{\partial T}{\partial t}\right|_i^k = \frac{T_i^{k+1} - T_i^k}{\Delta t} + \vartheta(\Delta t) \qquad (7)$$

The last expression (7) represents the progressive finite difference in time.

- The space derivative expression:

$$T_{i+1}^{k+1} = T_i^k + \Delta z \left.\frac{\partial T}{\partial z}\right|_i^k + \frac{1}{2}\Delta z^2 \left.\frac{\partial^2 T}{\partial z^2}\right|_i^k + \frac{1}{6}\Delta z^3 \left.\frac{\partial^3 T}{\partial z^3}\right|_i^k + \vartheta(\Delta z^4)$$

$$T_{i-1}^{k+1} = T_i^k - \Delta z \left.\frac{\partial T}{\partial z}\right|_i^k + \frac{1}{2}\Delta z^2 \left.\frac{\partial^2 T}{\partial z^2}\right|_i^k - \frac{1}{6}\Delta z^3 \left.\frac{\partial^3 T}{\partial z^3}\right|_i^k + \vartheta(\Delta z^4)$$

The sum of the two last above-mentioned expressions gives:

$$T_{i+1}^{k+1} + T_{i-1}^{k+1} = 2T_i^k + \Delta z^2 \left.\frac{\partial^2 T}{\partial z^2}\right|_i^k + \vartheta(\Delta z^4)$$

Either:

$$\left.\frac{\partial^2 T}{\partial z^2}\right|_i^k = \frac{T_{i+1}^k + T_{i-1}^k - 2T_i^k}{\Delta z^2} + \vartheta(\Delta z^2)\,. \qquad (8)$$

Equation (8) represents the finite difference centered on space. Therefore, by substitution of these finite quotients (7) and (8) in basic equation (4), we have:

$$\frac{T_i^{k+1} - T_i^k}{\Delta t} = \alpha \frac{T_{i+1}^k + T_{i-1}^k - 2T_i^k}{\Delta z^2} + \frac{2}{\rho C \pi d} Q - \frac{4}{\rho C d} h_{IV}(T_i^k - T_{IV})$$

Or, explicitly, the recurrence relation is:

$$T_i^{k+1} = \alpha \frac{\Delta t}{\Delta z^2}(T_{i+1}^k + T_{i-1}^k) + (1 - 2\alpha \frac{\Delta t}{\Delta z^2} - \frac{4h_{IV}}{\rho C d}\Delta t) T_i^k + \frac{2\Delta t}{\rho C d}(\frac{Q}{\pi} - 2h_{IV}T_{IV}) \qquad (9)$$

Equation (9) shows the temperature in point i at the moment $k+1$. It is obtained by the temperatures in this point and its immediate neighbours, at the previous moment (hereditary values).

B. Runge-Kutta method

To apply this method to the heat equation (4), it must be rewritten in the form of equation in the state space:

$$\dot{M} = M\theta + S = f(\theta_i, t_i)\,, \qquad (10)$$

where θ is the temperature matrix, M is the equivalent heat transfer coefficient matrix, S is the equivalent heat source matrix, and t is time. This method is traditionally called the semi-discretized method of lines. The convergence is better by choosing the sampling step and the most suitable time interval.

IV. RESULTS DISCUSSION

Simulation results were based on the numerical study of heat transfer equation with the (FDM) and the (RKM).

Figures 4 and 5 show the numerical simulation results in the case of FDM. We note that the temperature variation is symmetrical with respect to the roll center. The temperature remains practically stable in the medium, with a linear variation at the ends of the work rolls, caused by edge effects.

In the transition phase we observe, as time increases, curves which become increasingly round. The Fig. 5 shows more clearly the curve which becomes more accentuated when time grows.

Figures 6 and 7 describe the numerical simulation results of the thermal distribution along the work rolls by the Runge-Kutta method.

The same observations are illustrated on Fig. 6 where temperature is stable in the medium with a linear variation at the ends. Except, in this case, there is no curve even if time increases. We can detect better this effect on Fig. 7, where the temperature is always stable around the steady point 90°C.

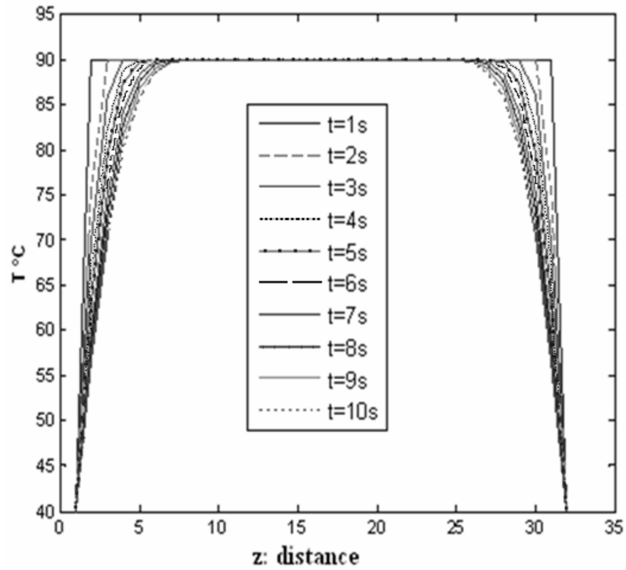

Fig. 4. Temperature distribution by the FDM.

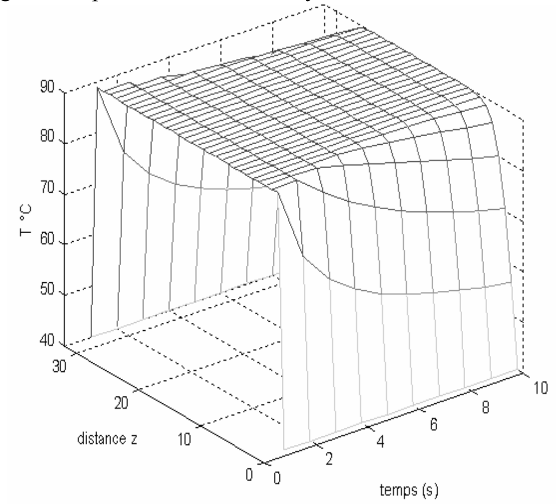

Fig.5. Temperature distribution by the FDM in 3D.

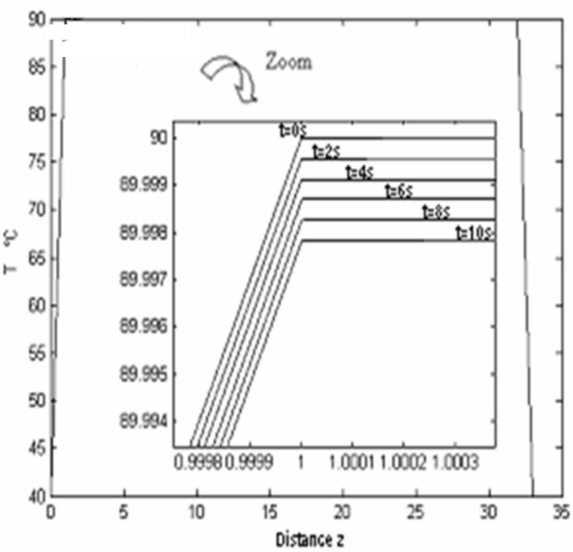

Fig.6. Temperature distribution by the RKM.

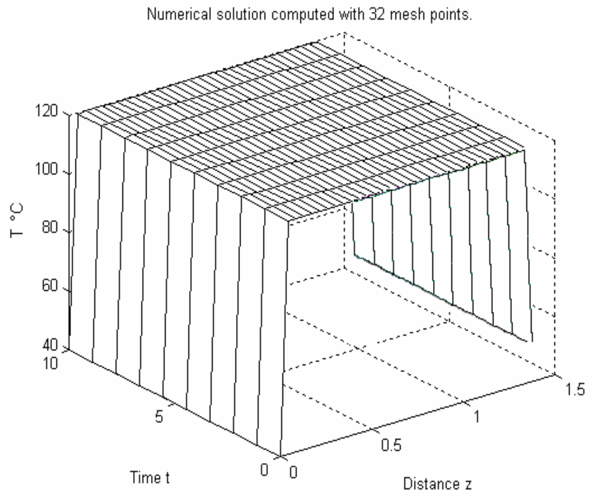

Fig. 7. Temperature distribution by the RKM in 3D.

The series of simulation results we carried out reflect partially the major aim of establishing the discretization of the heat transfer equation along the work rolls.

The key parameters that we evaluate in this article include: the thermal transfer coefficient by convection, the initial conditions and the boundary conditions.

V. CONCLUSIONS

In this paper we described two methods to calculate the thermal profile of work rolls. Numerical simulations are based on both recursive calculation methods and iterative methods. These latter are characterized by their rapidity calculation and results precision. Moreover, the development of the partial differential equations substantially reduces the storage memory and the computing time. The supposed boundary conditions have great effects on the temperature distribution along the work rolls. Indeed, the thermal profile development depends primarily on the cooling water flow. Thus, the cooling conditions (fluid temperature) and the corresponding heat transfer coefficients are very important in the model adjustment process. Although these parameters must be included in the identification stage in the following works, more precise and complete quality data are expected in the future. The thermal profile study of the work roll occurred in cold rolling is done by using physical bases and numerical developments described previously. Our perspective will be the development of a control law to reduce to the maximum the deformations of both the strip and the work rolls.

NOMENCLATURE

t (s) : time.

x, y, z (m) : coordinates in the direction of lamination, in the radial direction of the cylinder, in the direction of the length of the cylinder.

L (m): length of work rolls in the direction transverse z.

z_i : z at the i^{th} node, i=0,1,…,M.

T_i (°K): work roll temperature at the i^{th} zone.

α ($W.m^{-1}.K^{-1}$) : thermal diffusivity.

ρ ($Kg.m^{-3}$) : density of work roll material.

C (J.Kg^{-1}.K^{-1}): specific heat of the work roll.
d (m) : roll diameter.
Q_i (W.m^{-2}): equivalent total heat flux of the ith zone, i=1,…,M.
M: total cooling zone number along of work roll.
h_{IV} (W.m^{-2}.K^{-1}) : convective heat transfer coefficient at zone 4 (entry side of the mill)
h_{III} (W.m^{-2}.K^{-1}) : equivalent convective heat transfer coefficient at zone III (contact zone with the back-up roll).
h_{II} (W.m^{-2}.K^{-1}) : convective heat transfer coefficient at zone II, (exit side of the mill).
T_{IV} (°K): zone 4 coolant temperature (entry side) along of work roll.
T_0 (°K): work roll initial temperature.
T_{II} (°K) : zone II coolant temperature (exit side).
θ_{II} (rad) : zone II contact angle (entry side).
θ_{III} (rad) : contact angle between rolls.
θ_{IV} (rad) : zone IV contact angle (exit side).
q (W.m^{-2}.rad^{-1}): total heat flux due to rolling in the roll bite.
r (m) : work roll radius.

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