CONTROL OF A PRODUCTION-INVENTORY SYSTEM USING A PID AND DEMAND PREDICTION BASED CONTROLLER

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Abstract — The need of reducing inventory levels as much as possible without losing sales opportunities is an important goal not only for small but also for mid-size and large companies, on account of the high costs associated with large inventory stocks. In general, the performance of inventory systems is also affected by the Bullwhip effect caused, among other factors, by non-zero lead times. This paper proposes an automatic pipeline feedback order-based production control system (APIOBPCS) considering a demand with cyclic and stochastic components. The dynamics and delays of the production process are modeled as a pure delay. The control system structure consists of a PID controller and demand prediction based on an Extended Kalman Filter (EKF). The main objective of the controller is to stabilize and regulate the inventory levels about a desired set-point. The extended Kalman filter estimates the parameters of a Volterra time-series model to predict future values of the demand. The control system is evaluated by simulations, showing a good performance and better results than those achieved by using traditional inventory control techniques.

Keywords — Production-inventory systems, control, prediction, Extended Kalman Filter.

I. INTRODUCTION

Until recently, production and sales managers used to control inventory levels by means of two powerful but limited tools: intuition and experience. However, the size and complexity of modern production and sale operations have grown in such a way that it is not anymore convenient to regulate stock levels without having a quantitative assessment of the involved factors. This means making a smart or optimal decision not only on what or how much to order, but also, on when to do it.

Inventories are resources needed for production or commercialization processes, that are kept idle, waiting to be used when necessary. These resources can be of any kind: men, machines, raw material, money, graduates from the educational system, etc. Inventories are used to compensate the imbalances of the normal sequence of activities in production and sales processes. In other words, inventories should have a stabilizing effect on material flow patterns (Disney and Towill, 2003). Due to the fact that demand is usually unknown and stochastic in nature, it is not an easy task to keep inventories on an appropriate and regular level. If the level is set too low, maintenance costs may go down: there is no need for large storage spaces, the insurance costs decrease; devaluation costs are lower. However, there persists the risk of losing sales when demand grows beyond the expected figures. On the other hand, if inventory levels are kept too high, maintenance cost are usually higher: the volume of resources kept in stock increases, the required space is larger, and the devaluation and maintenance costs are higher. Therefore, an effective supply chain should be managed with an aim at keeping a high level of customer satisfaction while minimizing costs and maximizing profits (Rivera and Pew, 2005). Results of savings achieved by best-in-class companies, as a result of improving their supply chain operations, amount 5-6% of sales (Simulation Dynamics, 2003).

Although research in this area is not novel, it was only recently when the control systems community have paid attention to this topic. This is described thoroughly in the excellent revision of Ortega and Lin (2004). Previous research works have proposed systems to stabilize the inventory level as is the case of the works of John et al. (1994) and Disney and Towill (2003). More recently, the works of Grubbström and Wikner (1996), Samata and Al-Arami (2001) and Rivera and Pew (2005) have explicitly included dynamic controllers, such as PID, on the supply chain, and have obtained promising results. The present work proposes a simple control system whose main objective is to keep the inventory at a desired level in spite of fluctuations in the demand, taking into account lead times of the production system. The controller is based on an APIOBPCS model and uses a PID controller and demand prediction, to keep sta-
tionary inventory levels. Therefore, the inventory level set-point can be lowered without losing sales opportunities. It is assumed that demand signal has two components: a cyclic one and a Poisson stochastic perturbation. The demand is predicted by a dual joint EKF, which identifies the parameters of a Volterra model used to model it.

II. PRODUCTION-INVENTORY MODEL

The underlying theory of the open-loop model to describe inventory systems is briefly explained below. The dynamics of an inventory system is represented by a simple difference equation:

\[ i_{(k+1)} = i_{(k)} + o_{(k-\tau)} - d_{(k)} \]  

where \( i_{(k)} \) is the net inventory level, \( \tau \) represents the order fulfillment time, \( o_{(k-\tau)} \) represents the prior orders made \( \tau \)-days before, and \( d_{(k)} \) is the demand signal. The order \( o_{(k)} \) is generated by a reorder policy.

Traditionally, reorder policies have been based on Economic Order Quantity (EOQ) approaches:

- \((\hat{s}, Q)\) policy - When the inventory level reaches level \( \hat{s} \), \( Q \) units are ordered.
- \((\hat{s}, \hat{S})\) policy - When the inventory level becomes equal to or less than \( \hat{s} \), order up to the level \( \hat{S} \).

Periodic review systems

- \((nQ, \hat{s}, R)\) policy - If at a review time, the inventory level is less than or equal to \( \hat{s} \), a multiple \( nQ \) is ordered \((n = 1, 2, 3 \ldots)\); where \( n \) is chosen so that inventory reaches a level in the interval \([\hat{s}, \hat{s}+Q])\).
- \((\hat{S}, R)\) policy - At each review time, a sufficient quantity is ordered to bring the level of the inventory up to \( \hat{S} \).
- \((\hat{s}, \hat{S}, R)\) policy - If, at a review time, the inventory is less than or equal to \( \hat{s} \) a sufficient quantity is ordered to bring the level up to \( \hat{S} \).

EOQ approaches are widely used but they are not efficient enough, mainly because they do not have into account demand fluctuations.

On the other hand, APIOBPCS models have shown to perform well, stabilizing the system and reducing the bullwhip effect. Bullwhip effect refers to the scenario where orders to the suppliers tends to have larger fluctuations than sales to the buyer and this distortion propagates and amplifies itself when going upstream (Disney and Towill, 2003). A basic production-inventory systems based on the APIOBPCS scheme have four main components: the inventory, that can be modeled as an integrator, the production process that has been modeled in this paper as a finite time delay, the reorder policy and the demand predictor.

In addition, there are four fundamental information flows (Grubbström and Wikner, 1996), namely demand, inventory level, work-in-progress (WIP) and demand prediction \((\hat{d})\). Most of the order decision rules are based on one or more of these flows. That is:

\[ o_{(k)} = f[i_{(k)}, \hat{d}, WIP_{(k)}] \]  

These components are shown in Fig 1.

![Basic production-inventory model](image)

Figure 1: Basic production-inventory model.

In this work, demand is suppose to be cyclic, simulating a seasonal demand, with a stochastic component given by the addition of Poisson noise. For simplicity, a fixed order fulfillment time is assumed.

In contrast to the APIOBPCS analyzed in Disney and Towill (2003), where a filter acts as demand estimator and there is not a dynamical controller, our approach includes in the control loop a PID controller and the demand prediction generated by a joint dual EKF. We call this approach a PID-APIOBPCS model.

III. DEMAND PREDICTION

A. Volterra Models

Demand over time can be thought as a time-series, represented by a nonlinear autoregressive model. One way to model it is by means of a Volterra equation. The finite-dimensional discrete-time Volterra model used in this paper is a single-input, single-output model, relating an input sequence \(\{u(k)\}\), to an output sequence \(\{y(k)\}\) (Doyle et al., 2002). This relationship is defined by the equations:

\[ y_{(k)} = y_{0} + \sum_{n=1}^{N} \gamma_{n} M_{(k)} \]  

\[ \gamma_{n} M_{(k)} = \sum_{i_{1}=0}^{M} \cdots \sum_{i_{n}=0}^{M} \theta_{n}(i_{1}, \ldots, i_{n}) w(k-1) \cdots w(k-i_{n}) \]  

where \(y_{0}\) and \(\theta_{n}\) are the model parameters. It is convenient to introduce the notation \(V_{(N,M)}\), where \(N\) denotes the nonlinear degree of the model and \(M\) denotes its dynamic order. In our particular case a \(V_{(1,30)}\) model is used. Changing the name of the input and output signals, and taking the mentioned values for \(N\) and \(M\), Eq. 3 is reduced to:

\[ \hat{d}_{(k)} = d_{0} + \sum_{i=1}^{30} \theta_{i} d_{(k-i)} \]  

where \(d_{0}\) and \(\theta_{i}\) are the model parameters, \(\hat{d}_{(k)}\) is the actual estimated demand and \(d_{(k-i)}\) are past values of
the demand. The values of the unknown parameters will be found by a Kalman Filter, as shown in the next subsection.

B. Joint Extended Kalman Filter

The Kalman filter is characterized by a set of equations that synthesizes an optimal estimator of predictor-corrector type in the sense of minimizing the estimate error covariance \( \mathbf{P}_k \). In this particular case, a Joint Extend Kalman Filter (Ljung, 1979; Wan and van der Merwe, 2000) was used to solve the dual problem of simultaneously estimating the state and the model parameters \( \theta \) from the noisy demand signal.

To make the Volterra time-series into a Markovian process its necessary to model the demand given by a the Volterra equation Eq. 4 as that given by the nonlinear auto-regression system Eq. 5.

\[
\mathbf{x}_k = \mathbf{F} \mathbf{x}_k - 1, \mathbf{\theta}_k - 1 + \mathbf{B} \mathbf{\nu}_{k - 1}
\]

\[
y_k = \mathbf{C} \mathbf{x}_k + \mathbf{\eta}_k
\]

Then the model is rewritten as the state space system of Eq. 6:

\[
\begin{bmatrix}
  x_k \\
  x_{k-1} \\
  \vdots \\
  x_{k-M+1}
\end{bmatrix}
= \begin{bmatrix}
  f(x_{k-1}, \ldots, x_{k-M}, \mathbf{\theta}) \\
  1 \\
  0 \\
  \vdots \\
  0
\end{bmatrix}
\begin{bmatrix}
  x_{k-1} \\
  \mathbf{\theta}
\end{bmatrix}
+ \begin{bmatrix}
  \nu_k \\
  0 \\
  \vdots \\
  0
\end{bmatrix}
\]

\[
y_k = \begin{bmatrix}
  1 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
  x_k \\
  \mathbf{\theta}
\end{bmatrix}
+ \mathbf{\eta}_k
\]

where \( f(x_{k-1}, \ldots, x_{k-M}, \mathbf{\theta}_{k-1}) \) is the mentioned Volterra model, and \( \nu \) and \( \mathbf{\eta} \) are the process and measurement noises respectively. The output \( y_k \) is the estimated demand \( \hat{d}_k \) and the elements of the state vector \( \mathbf{x}_k \) are the past values of the demand. The joint EKF approach to determine the unknown parameters \( \mathbf{\theta} \) consists in augmenting the state vector \( \mathbf{x} \) with the parameter vector \( \mathbf{\theta} \). By doing this, a new state vector \( \mathbf{z}_k \) is obtained. Then, estimation is done recursively by writing the state-space equations for the joint state as

\[
\begin{bmatrix}
  \mathbf{x}_k \\
  \mathbf{\theta}
\end{bmatrix}
= \begin{bmatrix}
  \mathbf{F} \mathbf{x}_k - 1, \mathbf{\theta}_k - 1 \\
  0
\end{bmatrix}
+ \begin{bmatrix}
  \mathbf{B} \\
  0
\end{bmatrix}
\begin{bmatrix}
  \nu_k \\
  0
\end{bmatrix}
\]

\[
y_k = \begin{bmatrix}
  1 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
  \mathbf{x}_k \\
  \mathbf{\theta}
\end{bmatrix}
+ \mathbf{\eta}_k
\]

and running a EKF on the joint state-space to produce the simultaneous estimates of the states \( \mathbf{x}_k \) and \( \mathbf{\theta} \). The EKF equations can be synthesized as follow (Wan and Nelson, 2000). Initialize with:

\[
\hat{z}_0 = E[\mathbf{z}_0] \\
\mathbf{P}_z_0 = E[(\mathbf{z}_0 - \hat{z}_0)(\mathbf{z}_0 - \hat{z}_0)^T]
\]

where \( E \) means the expected value. Then, for \( k = 1, \ldots, \infty \), the time update equations of the EKF are:

\[
\mathbf{\hat{z}}_k = \mathbf{F}_k \mathbf{\hat{z}}_{k-1}, \mathbf{\theta}_{k-1}, \mathbf{\nu}_{k-1}
\]

\[
\mathbf{P}_z_k = \mathbf{A}_k \mathbf{P}_z_{k-1} \mathbf{A}_k^T + \mathbf{R}
\]

and the measurement update equations are:

\[
\mathbf{K}_k^* = \mathbf{P}_z_k \mathbf{C}_k \mathbf{C}_k^T (\mathbf{C}_k \mathbf{P}_z_k \mathbf{C}_k^T + \mathbf{R})^{-1}
\]

\[
\mathbf{\hat{z}}_k = \mathbf{\hat{z}}_{k-1} + \mathbf{K}_k^* (y_k - \mathbf{C} \mathbf{\hat{z}}_k)
\]

\[
\mathbf{P}_z_k = (1 - \mathbf{K}_k^* \mathbf{C}_k) \mathbf{P}_z_k
\]

with \( A = \frac{\partial F_k(z, \theta, \nu)}{\partial z} \mid z_k \)

In the EKF equations, \( F_k(\cdot) \) stands for the new joint model of Eq. 7, \( \mathbf{P}_z \) is the estimate error covariance, \( \mathbf{K}_k^* \) is the Kalman gain and \( \mathbf{R} \) are the process and measurement noise covariance respectively.

Once the model parameters \( \hat{d}_k \) and \( \theta \) have been estimated, they are used together with the model to get a prediction of on step ahead. This predicted state vector is then used for the PID-APIOBPCS reorder policy.

IV. PROPOSED CONTROLLER

As mentioned, the control approach proposed in this paper is to use some kind of APIOBPCS, namely PID-APIOBPCS.

APIOBPCS has the main advantage of including in the decision rule the value of the WIP. A basic diagram of a PIOBPCS model is shown in Fig. 2. In this case, the reorder policy equations are presented in Eq. 11:

\[
\begin{align*}
\hat{d}_k &= \hat{d}_k + \hat{\nu}_{ref}(k) - \nu_{ref}(k) + dWIP_{k-1} - WIP_{k-1}
WIP_k &= WIP_{k-1} + \nu_k - \nu_{k-1}
\end{align*}
\]

\[
dWIP_k = T_{p} \hat{d}_k
\]

where, \( \hat{d}_k \) is the estimated demand, and \( \hat{\nu}_{ref}(k) \) is the inventory level reference. Constant \( T_{p} \) is related to the time to adjust the inventory level, \( T_{w} \) is the estimate of the production lead time, and \( T_{w} \) is the time needed to adjust WIP.

Figure 2: Ordering system incorporating WIP feedback.

On the other hand, the approach of using only a PID as suggested in Grubbstöm and Wikner (1996), and in Rivera and Pew (2005) to model an order decision rule does not involve an explicit forecasting unit to estimate demand. So, fusing both controllers, a new and more complete controller is obtained. The proposed control schema is shown in Fig. 3.
Parameters

Eq. 7, and equations of the EKF, Eq. 8, 9 and 10.

In a Matlab S-Function, using the model presented in
controller a Matlab-Simulink model was mounted. The
to evaluate the performance of the proposed con-
negative values do not have a real meaning for inven-
tories when orders are greater than 200 and order with
because we assume that the production system satu-
reorder policies are physically limited: control actions
and WIP compensation. In addition, we use a PID
PID controller, using memory of past results (integral
term) and anticipating trends (derivative term) as well
as a proportional term for their future decisions.

To Kunreuther (1969), top level managers are found
to act in a three-terms-control mode, similarly to a
PID controller as decision rule. It is worth to note that
a PID controller is not a capricious choice. According
to Volterra model became stable and the demand signal
ter a short transitory, the coefficients of the identified
and Fig. 7. This is an acceptable policy for controlling
demand.

After the Simulink model was set, the joint EKF was
tested. Figures 4 and 5 show the estimated parameters
and the predicted demand. As these figures show, af-
methods, with the advantages of a PID action. The inclusion
of a PID controller is not a capricious choice. According
to Kunreuther (1969), top level managers are found
to act in a three-terms-control mode, similarly to a
PID controller as decision rule. It is worth to note that
reorder policies are physically limited: control actions
should not take values above 200 or below 0. That is
because we assume that the production system saturates
when orders are greater than 200 and order with
negative values do not have a real meaning for inven-
tory systems.

V. SIMULATION RESULTS

To evaluate the performance of the proposed con-
troller a Matlab-Simulink model was mounted. The
joint Dual Extended Kalman filter was implemented in
a Matlab S-Function, using the model presented in
Fig. 3: Proposed PID-APIOBPCS controller.

\[
o(k) = a(k-1) + K_p[e(k) - e(k-1)] + K_i e(k-1) + K_D [e(k) - 2e(k-1) + e(k-2)]
\]

Equations 12 represent the reorder policy for the PID-
APIOBPCS case. As it can be seen the reorder policy
involves the same variables as the APIOBPCS method,
and were set to 10 and 40 respectively.

The model to be estimated by the EKF is the one
presented in Eq. 4. Demand signal was simulated by a
sum of \( \sin \) and \( \cos \) terms of different amplitude, phase
and frequency. Poisson noise with \( \lambda = 10 \) was also
added. This noise has also a negative component, so variations on demand can be positive or negative. For all simulations, the inventory level set-point has been
set to 20 units and the PID action has been limited
to a maximum of 200 units assuming that this is the
capacity of the production system. In addition, an
extra term has been added to the demand signal to
represent sudden stochastic changes on the value of
demand.

\[
WIP_{(k)} = WIP_{(k-1)} + a(k) - a(k-\tau)
\]

\[
dWIP_{(k)} = d_{(k)}
\]

B. PID reorder policy

The next step was to test a production-inventory sys-
tem with a PID reorder policy but without demand
forecast and WIP feedback. PID parameters for this
experiment were set to \( K_p = 0.5, K_i = 0.1, K_D = 0.1 \).
These values were found by heuristic and the criterion for the selection of these values were to obtain a smooth but fast response to demand and inventory set-point level variations. As in the previous experiment, inventory level set-point was 20 units and the maximum control action available was 200 units. In this case, simulation was run for 730 days (2 years). Results are presented in Fig. 8 and Fig. 9. For this experiment, results are a little bit better: bullwhip effect is not as obvious but it can be seen in those peaks that appear in Fig. 9. In this case, however, the inventory level is still oscillatory and falls below zero too many times. In addition, when demand changes abruptly, the system present large positive and negative peaks of around 10-day length.

C. APIOBCS reorder policy

The already mentioned APIOBCS method was also tested. Gain values were all set to one, due to the fact that these values are related to production and lead times, which were suppose to be one in this work. Desired inventory level was set to 20. Results are presented in Fig. 10 and 11. These figures show a good performance. The inventory level stays stable around 20 units and compared to the previous case, inventory level seldom falls below 0. The bullwhip effect is appreciated in some peaks, caused by abrupt changes in demand, but they are canceled in around 7 days.
D. PID-APIOBCS reorder policy

Finally, the proposed control system was tested. PID parameters were set to $K_P = 1.3$, $K_I = 1.1$, $K_D = 1$. As in Subsection B, these values were found by heuristics. In this case the criterion was to find values that result in a fast but damped response to demand and inventory set-point level variations. The desired inventory level was, again, set to 20 units. Results are presented in Fig. 12 and 13. As it can be seen on these figures, the inventory level is more stable. Peaks still exist, but they are smaller and are canceled in around 5 days. Bullwhip effect is almost the same than in the previous PID and APIOBCS cases. Although there are some inventory level values below zero, these are not as many as in previous experiments. So, when comparing the different experiments, it can be said that to keep an acceptable level of lost sales, desired inventory level should be higher in the case of using an EOQ policy, or only a PID controller, or with the APIOBCS method.

VI. CONCLUSIONS AND FUTURE RESEARCH

In this paper a reorder policy based on an APIOBCS model was proposed. The new reorder policy includes a PID controller and an estimate of the demand made with a joint dual EKF. The performance of the proposed controller was tested in simulations. The proposed policy was compared to classical reorder policies such as a $(\hat{s}, \hat{S})$ policy, a PID based reorder policy and an APIOBCS policy. Simulation results show a good performance of the proposed controller that reduces the bullwhip effect and keeps lower costs than with other classical controllers. Future research will include more complex models for the production stage and the inclusion of optimum operative conditions in the controller as well as in the desired inventory level planning. The use of intelligent optimal controllers such as Adaptive Critic Designs is also an open field in this area. Control of multiple-echelon and multiple-products production-inventory systems could be addressed using this type of controllers.

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