Abstract—Vibrating plates may become strong sources of unwanted sound exposing humans to high levels of noise, particularly at low frequencies. Therefore, knowledge of the sound radiation of vibrating plates is very important for noise control engineering. This paper presents a method to estimate the sound power radiated from a baffled vibrating plate. Instead of using a modal radiation approach, the sound power is expressed in terms of volume velocities of a number of elemental radiators by dividing the vibrating surface into small virtual elements. Thus, the method discussed here is based on the radiation resistance matrix where its entries are calculated by treating each element as a circular piston having an area equal of that of the corresponding element. As practical examples, the method is applied to estimate the sound radiation characteristics of circular and rectangular plates. Numerical results indicate the accuracy and efficiency of the numerical technique.

Keywords—Plates, Sound radiation, Sound power, Noise control, Numerical methods.

I. INTRODUCTION

The sound radiation from a vibrating plate structure is of great practical importance in Applied Mechanics and has been an active research subject for many years. In particular, the sound radiation from vibrating plates commonly appears in industry, airplanes, cars, machinery, buildings, and electroacoustical devices. Knowledge of the sound radiation characteristics of these structures is important not only for the researcher who wishes to understand their behavior, but also for the engineer whose duty it is to prevent any harmful noise levels which may occur in the course of the industrial use of such structures.

In recent years, several studies have been committed to estimate the sound radiation characteristics of plates subject to different kinds of excitation and with various boundary conditions. When a plate is excited by an external force its vibration transfers energy to the surrounding fluid medium which is radiated as sound waves.

The basic equation for the calculation of the sound pressure field due to a closed body vibrating harmonically at circular frequency $\omega$, is obtained by a superposition of simple monopole and dipole sources distributed over the surface. This equation is known as the Kirchoff-Helmholtz integral equation (Pierce, 1989; Morse and Ingard, 1986)

$$p(r, \omega) = \iint_S \left[ p(r', \omega) \frac{\partial G(r|r')}{\partial n} + j\rho \omega V(r', \omega)G(r|r') \right] dS,$$

where $p(r, \omega)$ is the complex sound pressure at a point $r$ outside the surface, $\rho$ is the fluid density, $V(r', \omega)$ is the normal complex velocity amplitude at a point $r'$ on a structural surface of area $S$, $n$ indicates the outward normal direction perpendicular to the surface, $j = \sqrt{-1}$, and $G(r|r') = e^{jkr}/4\pi R$ is the freespace Green’s function, where $R = |r - r'|$ and $k$ is the acoustic wavenumber. This Green’s function is equivalent to the sound field of a simple source in an infinite fluid medium. For a vibrating flat plate set in an infinite baffle, the Green’s function satisfies the Neumann boundary condition on the surface and the Green’s function equals twice that of $G(r|r')$ in Eq. (1). Therefore, Eq. (1) can be written as Rayleigh’s second integral (Rayleigh, 1945)

$$p(r, \omega) = \frac{j\rho \omega}{2\pi} \iint_S V(\omega, r')G(r|r') dS.$$

In some cases, Eq. (2) must be evaluated by using the calculus of residues to overcome the singularity of $G(r|r')$ when $r \to r'$.

Although Eq. (2) has analytical solutions only for a few cases, the Rayleigh integral has been used as an alternative to the finite/boundary elements solution of an acoustic boundary problem. Applications of the Rayleigh integral to calculate the sound radiation from the structural modes of plates having different shapes and boundary conditions, have been reported by several authors (Kirkup, 1994; Wodtke and Lamancusa,
It can be noted that Eq. (2) gives the value of the sound pressure at a point \( r \). However, it is convenient to characterize the sound radiated from a vibrating structure by a single global quantity. Thus, the source strength is commonly characterized by the sound power radiated. The time-averaged sound power radiated by a baffled vibrating plate’s surface is defined as (Pierce, 1989)

\[
\bar{\Pi}_{\text{rad}}(\omega) = \frac{1}{2} \int_{S} \Re \{ p(r, \omega) V^*(r', \omega) \} dS, \tag{3}
\]

where \( S \) is the total surface area of the plate, \( r' \) are the coordinates of the point on the surface, \( p(r, \omega) \) is the complex sound pressure in the near field, and \( * \) denotes the complex conjugate. The product \( p(r, \omega) V(r', \omega) \) is called the normal sound intensity.

If the sound pressure is estimated from Rayleigh’s integral representation of Eq. (2), the evaluation of \( \bar{\Pi}_{\text{rad}} \) requires to solve a quadruple integral which greatly increases the computational demands of determining the sound power radiated.

When the normal vibration velocity distribution over the surface of the plate is known, two approaches have commonly been used to determine the sound power radiated: 1) to integrate the far-field sound intensity over an imaginary hemisphere enclosing the plate and 2) to integrate the sound intensity over the surface of the vibrating plate. In this paper, the second approach will be explored.

### II. MATRIX METHODS

#### A. Modal Radiation Resistance

When a plate is excited at a given frequency of excitation, the response of the plate is a superposition of a number of individual modal responses. Thus, the normal velocity \( V(r) \) at any location \( r \) on the structure can be expressed as the linear combination of in-vacuo modal contributions,

\[
V(r) = \sum_{i=1}^{\infty} V_i \psi_i(r), \tag{4}
\]

where \( V_i \) is the complex velocity amplitude of the \( i \)-th mode, and \( \psi_i(r) \) is the value of the associated mode shape function at location \( r \). An estimation of the velocity can be given by truncating the sum at a finite number \( N \) of modes chosen to ensure a satisfactory convergence of the series. Then, Eq. (4) can be expressed as

\[
V(r) = v^T \psi(r), \tag{5}
\]

where \( v^T = [V_1 V_2 \cdots V_N] \) is a column vector of complex modal velocities and \( \psi^T = [\psi_1 \psi_2 \cdots \psi_N] \) is a column vector of mode shape functions evaluated at location \( r \). Now, combining Eqs. (2), (3) and (5) we obtain

\[
\bar{\Pi}_{\text{rad}}(\omega) = v^T \mathbf{M} v, \tag{6}
\]

where \( H \) denotes the Hermitian (conjugate transpose), and \( \mathbf{M} \) is an \( N \times N \) symmetric positive definite matrix called modal radiation resistance whose entries are

\[
M_{ik} = \frac{\omega^2 p a}{4\pi} \int_{s} \int_{s} \psi_i^*(r') \sin k R \psi_k(r) dS' dS. \tag{7}
\]

The geometry is shown in Fig. 1.

![Figure 1: Geometry of the problem.](image)

\( M_{ik} \) is called the self-radiation modal resistance if \( i = k \) and the mutual modal resistance if \( i \neq k \). Physically, a self-radiation term measures the effectiveness of an individual mode in generating sound and a mutual radiation term quantifies cross-modal coupling, indicating how the sound field produced by one mode affects the vibration of another mode.

We can observe that the entries of matrix \( \mathbf{M} \) depend on frequency and plate’s geometry. It is important to notice, however, that the shapes of the structural modes change when different sets of boundary conditions are selected. Therefore, \( \mathbf{M} \) depends on the velocity distribution on the vibrating plate.

In addition, the evaluation of \( \mathbf{M} \) requires to solve quadruple integrals. Theoretical analysis of Eq. (7) has been presented for certain geometries and sets of boundary conditions, leading to approximate and asymptotic solutions.

In the case of simply-supported rectangular plates, \( \mathbf{M} \) is sparse since cross-modal radiation only occurs between a pair of modes with a similar index-type. Therefore, only a quarter of the matrix elements are non-zero (Snyder and Tanaka, 1995). Some authors have reported mathematical techniques to reduce the quadruple integrals to a finite summation of single integrals (Pierce et al., 2002) or double integrals (Li and Gibeling, 2000) each of which can be evaluated numerically. Li (2001) has expanded the integrand leading to an infinite summation that is convergent over all frequencies. More recently, Graham (2007) has presented analytical formulas obtained by using contour integration techniques for modal wavelength.
smaller than the acoustic wavelength and the plate dimensions. Rdzanek et al. (2007) have also presented low frequency approximated formulas for an elastically supported circular plate, considering its axisymmetric and asymmetric vibration modes. However, for general geometries and boundary conditions, Eq. (7) requires the use of numerical integration which may obviously be a computing-intensive task.

### B. Acoustic Radiation Resistance

Another approach consists in discretizing the plane vibrating structure into a set of elements. Then, the dimension of the problem is given by the number of virtual sub-divisions of the structure instead of the finite number of modes. This approach has been called the *lumped parameter model* which uses simplified representations of the particle velocity distribution on the body surface and the acoustic radiation resistance matrix.

Associated to the radiation resistance matrix are the radiation modes. These modes are related with surface velocity distributions that produce orthogonal sound pressure radiation fields. The radiation mode technique has a number of parallels to the usual modal approach and thus many analogies can be formulated.

The origins of the radiation mode technique can be found in a seminal study by Borgiotti (1990) on the determination of the sound power radiated by a vibrating body submerged in a fluid from boundary measurements. This matrix method has been mainly used for (Arenas, 2008): 1) active structural acoustical control of low frequency noise radiated by vibrating structures and 2) to the application of structural optimization for passive noise control (Kessels, 2001; Maury and Elliott, 2005). A practical application of the acoustic radiation resistance matrix was presented by Paddock and Koopmann (1995) to assess the noise characteristics of machines. In addition, the combination of the acoustic radiation resistance matrix and the measurement of volume velocity by means of accelerometers on the surface of a rectangular plate has produced accurate prediction of the sound radiation efficiency (Arenas and Crocker, 2002).

*Lumped parameter model*

Fahnline and Koopmann (1996) defined a model for the acoustic radiation from a vibrating structure by dividing its surface into elements, expanding the acoustic field from each of the elements in a multipole expansion, and then truncating all but the lowest-order terms in the expansion. The basis functions for the numerical analysis were taken as the acoustic fields of discrete monopole, dipole, and tripole sources located at the geometric centers of the surface elements.

Considering that the vibrating structure surface $S$ is divided into $N$ small elements of area $S_k$, with $k = 1, 2, ..., N$, the local specific acoustic radiation impedance is defined as the complex ratio between the sound pressure amplitude $p_i$ at point $i$ due to a point source located at point $k$, and the normal velocity $V_k$

$$Z_{ik} = \frac{p_i}{V_k} \bigg|_{on \ S} .$$  (8)

Assuming that the characteristic length of the surface elements is small compared to a typical acoustic wavelength, then the pressure and velocity can be considered constant over each element and can be represented by an average value. Therefore, Eq. (8) is

$$Z_{ik} = \frac{p_i}{u_k} .$$  (9)

where $u_k = \int V_k dS_k$ is the volume velocity at point $k$, and the acoustic radiation resistance matrix $R_{ik}$ can be obtained by taking the real part of $Z_{ik}$.

Thus, combination of Eqs. (3) and (9), yields (Paddock and Koopmann, 1995)

$$\bar{\Pi}_{rad} = \frac{1}{4} \sum_{i=1}^{N} \sum_{k=1}^{N} Z_{ik} u_i u_k^* + \frac{1}{4} \sum_{i=1}^{N} \sum_{k=1}^{N} Z_{ik}^* u_i^* u_k .$$  (10)

Now, interchanging the indices and order in the summations and after the application of the principle of reciprocity to acoustic fields $Z_{ik} = Z_{ki}$ (Pierce, 1989), we obtain

$$\bar{\Pi}_{rad}(\omega) = \frac{1}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} R_{ik} u_i u_k^* = \frac{1}{2} \mathbf{u}^T \mathbf{R} \mathbf{u} ,$$  (11)

where $\mathbf{u}$ is the $N \times 1$ complex volume velocity vector, $H$ denotes the Hermitian, and $R_{ik} = \Re{Z_{ik}}$ are the elements of the real $N \times N$ acoustic radiation resistance matrix $\mathbf{R}$.

Now, if we write the elements of $\mathbf{u}$ as $u_i = |u_i| e^{j\phi_i}$, then (Arenas and Crocker, 2002)

$$\bar{\Pi}_{rad}(\omega) = \frac{1}{2} \sum_{i=1}^{N} R_{ii} |u_i|^2$$

$$+ \frac{1}{2} \sum_{i=1}^{N} \sum_{k=1}^{N} R_{ik} |u_i| |u_k| \cos(\phi_k - \phi_i) .$$  (12)

The first part on the right of Eq. (12) corresponds to the self-resistance and the second part is the cross-resistance, which gives a measure of the total acoustic coupling between the $i$th and $k$th surface elements.

From Eq. (11), we can observe that $\bar{\Pi}_{rad}$ is a positive quantity, for all $\mathbf{u} \neq 0$, so that the matrix $\mathbf{R}$ is also a symmetric positive definite matrix.

*Evaluation of $R_{ik}$*

A disadvantage of the lumped parameter model as presented by Fahnline and Koopmann (1996) is the amount of computational time spent in solving the system of equations for the source amplitudes. A method
to overcome this difficulty is to treat each surface element on the plate’s surface as a circular piston having an area equal to that of the corresponding element, as shown in Fig. 2.

Using this approach, the values for the self-resistance are well-known and are given by (Pierce, 1989)

\[ R_{ii} = \rho c S_i \left[ 1 - \frac{J_1(2ka_i)}{ka_i} \right], \]

(13)

where \( c \) is the speed of sound in the fluid, \( J_1 \) is the first-order Bessel function, \( S_i \) is the surface of the equivalent piston, and \( a_i = \sqrt{S_i/\pi} \) is the radius of the piston.

There are no analytical expression for the cross-resistances, but Stepanishen (1978) has presented the following approximate result

\[ R_{ik} = \frac{2\rho c k^2 S_i S_k}{\pi} \left[ \frac{J_1(ka_i)}{ka_i} \right] \frac{\sin kr_{ik}}{kr_{ik}}, \]

(14)

where \( S_i \) and \( S_k \) are the surfaces of the equivalent pistons, \( a_i = \sqrt{S_i/\pi} \) and \( a_k = \sqrt{S_k/\pi} \) are the radii of the pistons, and \( r_{ik} \) is the distance between the center points of each piston.

There are two important observations about \( R \): 1) it depends only on the frequency and geometry of the plate and 2) it can be numerically handled as an hypermatrix allowing storage for reusing. This increases time-efficiency and can make the solution of significantly larger problems feasible. In addition, Eqs. (13) and (14) can be easily implemented into computer codes.

**Sound radiation efficiency**

A nondimensional parameter is commonly used to relate calculations of the mean square normal velocity, averaged over the radiating surface, and the radiated power. It is called the sound radiation efficiency, or radiation ratio, and is defined as (Pierce, 1989)

\[ \sigma_{rad} = \frac{\bar{\Pi}_{rad}}{\rho c S \langle V^2 \rangle}, \]

(15)

where the radiated sound power \( \bar{\Pi}_{rad} \) is normalized by the product of the radiating surface area \( S \), the characteristic impedance of the fluid \( \rho c \), and the space-averaged mean square normal vibration velocity amplitude \( \langle V^2 \rangle \) defined as

\[ \langle V^2 \rangle = \frac{1}{2S} \int_S |V|^2 dS. \]

(16)

By analogy with a lumped element system, the equivalent specific radiation resistance is \( \rho c \sigma_{rad} \).

**III. NUMERICAL RESULTS**

To give some applications of the matrix method discussed above, numerical simulations were carried out for two cases: an annular guided plate and a rectangular plate.

**A. Annular Guided Plate**

The guided thin annular plate is used here since the structural vibration of this kind of plate is simple to express into mathematical formulae. In addition, study of annular plates is of practical importance to determine the noise radiated from many mechanical or structural components in automotive industry (Gerges, 2004).

The annular plate has an external radius \( a \) and internal radius \( b \) [total surface \( \pi(a^2 - b^2) \)], with aspect ratio \( \gamma = a/b \).

To simplify the implementation of the computational codes, the annular plate is divided into small elements of equal area. Transformation to a classical polar coordinate system will produce surface elements of different area. Using a discretization process similar to the one used for a circular disk (Arenas, 2008), the surface is divided into equally spaced concentric rings where each ring is subsequently divided into elements of equal area, as shown in Fig. 3. The crosses in Fig. 3 indicate the center point of each element.

![Figure 3: Discretization of the annular plate into 432 elements of equal area.](image-url)

The plate was divided into \( M = 432 \) elements. This discretization produces a \( 432 \times 1 \) vector \( \mathbf{u} \) and
radiation efficiency about four times faster than numerical integration are shown for comparison. It is observed that the method presented here computes the numerical results compare quite well with the theory for $ka \leq 4$.

Consequently, Eq. (16) can be estimated by

$$\langle V^2 \rangle = \frac{M}{2\pi^2 (a^2 - b^2)^2} \sum_{j=1}^{M} |u_j|^2$$

(17)

$$= \frac{M}{2\pi^2 (a^2 - b^2)^2} u^H u.$$  

The axisymmetric free vibration of an undamped thin guided annular plate, assuming light fluid loading (air), is represented by the normal velocity mode function (Rdzanek, 2003)

$$V_n(r) = V_{0m} \left\{ J_0(k_n r) - \frac{J_1(\beta)}{Y_1(\beta)} Y_0(k_n r) \right\},$$

(18)

where $r$ is the radial distance from the plate center, $V_{0m}$ is the complex velocity amplitude of an axisymmetric mode $m$, $J_0$ and $J_1$ are Bessel functions of the first kind, and $Y_0$ and $Y_1$ are modified Bessel functions of the second kind, $k_n = k_p = \beta / b$ is the structural wavenumber, and $\beta = b \sqrt{\nu (\rho_s / B)}^{1/4}$ are the roots of the frequency equation $J_1(\gamma \beta) Y_1(\beta) - J_1(\beta) Y_1(\gamma \beta) = 0$, where $\rho_s$ is the mass density per unit area of the plate, $B = Eh^3/12(1 - \nu^2)$ is the bending stiffness of the plate, $h$ is the thickness of the plate, $E$ is Young's modulus and $\nu$ is Poisson's ratio.

Figure 4 shows the results for the radiation efficiency of the first four axisymmetric modes of a guided annular plate of aspect ratio $\gamma = 2$ predicted by using Eqs. (11), (15), (17) and (18). In addition, the results obtained by numerical integration are shown for comparison. It is observed that the method presented here computes the radiation efficiency about four times faster than numerical integration for models of this size. The results are plotted as a function of the dimensionless ratio between the acoustic wavenumber $k$, and the structural wavenumber $k_p$. Thus, $k/k_p$ is a measure of the critical frequency, at which the propagation speed of the bending wave in the plate equals the speed of sound in the air. Therefore, when $k/k_p < 1$ (subsonic wave) the mode is below the critical frequency and when $k/k_p > 1$ (supersonic wave) it is above.

The results in Fig. 4 are in good agreement with those presented by Rdzanek (2003) which he determined from complex integration and asymptotic approximations. Differences are appreciated close to the critical frequency. Clearly, this is because the method is discrete in nature. It has to be noticed that the curves shown in Fig. 4 are applicable when the annular plate response is dominated by one mode, that is to say, at resonance. The following example will explore off-resonance conditions.

B. Rectangular Plates

Many research has been conducted in structural acoustics using simply-supported rectangular flat plates, since they exhibit all the relevant physical features and this problem admits an analytical description of the modes. Consider a simply-supported, rectangular, flat plate, of length $a$ and width $b$, which is mounted in an infinite rigid baffle. If the plate is thin enough to obey the bending wave equation, the normal velocity at any location $(x, y)$ on the surface of the plate is

$$V(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin(k_m x) \sin(k_n y),$$

(19)

where $A_{mn}$ is the complex velocity amplitude of the mode $(m, n)$, $k_m = m \pi / a$ and $k_n = n \pi / b$. If the summations in Eq. (19) are truncated to a finite number of modes $M \times N$, we can express Eq. (19) in matrix form as

$$V(x, y) = \alpha^T(x) \beta(y),$$

(20)

where $\alpha(x)$ is an $M \times 1$ vector of elements $\alpha_m = \sin(k_m x)$, $\beta(y)$ is an $N \times 1$ vector of elements $\beta_n = \sin(k_n y)$, and $\alpha$ is an $M \times N$ matrix of complex velocity amplitudes.

Now if the plate is excited by a harmonic point force concentrated at the point $(x_0, y_0)$, the entries of matrix $\alpha$ are

$$A_{mn} = \frac{j \alpha \omega F}{M} \frac{\alpha_m(x_0) \beta_n(y_0)}{\omega_m^2 (1 + j \eta) - \omega^2},$$

(21)

where $\omega$ is the frequency of the point force, $F$ is the amplitude of the point force, $M$ is the total mass of
the plate, η is the damping loss factor of the plate, and ω_{mn} is the natural frequency of mode \((m,n)\) given by

\[
\omega_{mn} = (B/\rho h)^{1/2} \left[ k_m^2 + k_n^2 \right].
\]  

(22)

For a rectangular plate divided into \(M\) equal small elements, Eq. (16) can be now estimated by

\[
\langle V^2 \rangle = \frac{M}{2(ab)^2} \sum_{j=1}^{M} |u_j|^2 = \frac{M}{2(ab)^2} u^H u.
\]  

(23)

Based on Eq. (11), the sound radiation efficiency of a simply-supported rectangular aluminum plate \((E = 7.1 \times 10^{10} \text{ N/m}^2, \rho = 2700 \text{ kg/m}^3)\) is calculated as an example. The plate’s surface is \(0.5 \times 0.6 \text{ m}^2\) and its thickness \(h = 3 \text{ mm}\). In addition, to excite a relatively large number of modes, location of a unit point excitation force is fixed at \((x = 0.02 \text{ m}, y = 0.02 \text{ m})\) relative to the lower left corner, and the frequency is varied. The plate was divided into \(20 \times 20\) elements \((M = 400)\).

To give an indication of the effects of increasing plate’s internal damping, damping loss factors η = 0.1, 0.2 and 0.4 were chosen. The infinite numbers of modes in Eq. (19) are truncated to a finite number. For this example, all modes with natural frequencies below 10 kHz are included in the computation. Figure 5 presents the results for sound radiation efficiency from Eq. (11) as a function of frequency. As a reference, the values of the frequency of the first resonance \(f_{11}\) and the critical frequency \(f_c\) have been indicated in Fig. 5. The radiation efficiency obtained for the first mode is also plotted for comparison.

Figure 5: \(\sigma_{\text{rad}}\) for a simply-supported aluminum plate \((0.5 \times 0.6 \times 0.003 \text{ m}^3)\) for different values of damping loss factor.

Again, the results of the numerical evaluation are in good agreement with the results presented by Xie et al. (2005) that were calculated for the same plate by using a modal summation approach. We can observe that below 70 Hz the radiation efficiency of the first mode determines the overall result. This is due to the dominance of this mode in the response. Above this frequency the results drop, only rising to unity close to \(f_c\) of about 4 kHz.

It can be seen that the radiation efficiency is independent of the damping loss factor for frequencies below \(f_{11}\) and well above \(f_c\). Between these frequencies, the radiation efficiency is proportional to the damping loss factor. In addition, the sound power level (reference power of \(10^{-12} \text{ W}\)) and the space-averaged normal vibration velocity level (reference velocity of \(10^{-9} \text{ m/s}\)) are shown in Fig. 6.

Figure 6: Sound power and space-averaged velocity level for a simply-supported aluminum plate \((0.5 \times 0.6 \times 0.003 \text{ m}^3)\) for different values of damping loss factor.

We can observe the influence of increasing of structural damping in reducing the vibration velocity and sound power level at resonances of a plate excited by a point force. Well below the first resonance the effect is quite limited. It has to be noted, however, that in practical applications the loss factor is a function of frequency and, in general, low values of damping are measured at high frequencies. Therefore, it is considered essential to determine the damping loss factor of the plate experimentally.

Now, the same plate with η = 0.1 was subjected to a fully-clamped boundary condition and the radiation efficiency was again evaluated numerically. The solution for the flexural vibration in a fully-clamped rectangular plate is more difficult than the simply-supported case. Here, the approximate solution presented by Arenas (2003) is used for all the computations. Figure 6 shows the comparison between the radiation efficiency for the simply-supported and clamped plates as a function of frequency.

We observe that, for low frequencies, \(\sigma_{\text{rad}}\) is not strongly affected when the boundary changes from the simply-supported to the clamped case. The simply-supported plate exhibits a radiation efficiency that is somewhat larger than the clamped plate. In the
medium frequency range the radiation efficiency is larger in the clamped case. Close to and above $f_c$, the radiation efficiency is not considerably affected by the boundary condition. The results agree well with those obtained using numerical integration of the Rayleigh’s integral (Berry and Nicolas, 1994).

In noise control engineering it is also important to select a proper material to make a plate acceptable silent for a given force and particular geometry. Then, to give an indication of the effects of changing plate’s material, consider now an undamped simply-supported square plate of side 0.6 m and $h = 6$ mm, excited by an arbitrarily harmonic point force of amplitude $F = 9$ N, fixed at $(x = 0.02$ m, $y = 0.02$ m) relative to the lower left corner. Consider that the plate is made of three different materials: polystyrene ($E = 3.6 \times 10^9$ N/m$^2$, $\rho = 1050$ kg/m$^3$, $\nu = 0.24$), hardened glass ($E = 72 \times 10^9$ N/m$^2$, $\rho = 2900$ kg/m$^3$, $\nu = 0.24$), and steel ($E = 210 \times 10^9$ N/m$^2$, $\rho = 7850$ kg/m$^3$, $\nu = 0.3$). The materials are the same as the ones used recently by Ržanek et al. (2007) to model a circular hatchway. Figure 8 shows the calculated sound power levels for low frequencies.

We observe that the polystyrene plate generates a sound power level that exceeds 70 dB near its first resonance around 30 Hz. The hardened glass and steel plates are much more silent up to 50 Hz, where the sound power level does not exceed 40 dB. We also observe that the lowest vibration modes of the plates generate considerable acoustic noise and the polystyrene plate is the most unfavorable choice of the three plates if low frequency noise have to be kept to a level that would be acceptable from a comfort aspect or design requirement.

IV. CONCLUSIONS

This article discussed a matrix method to numerically estimate the sound power radiated from planar vibrat-


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