EFFICIENCY MEASUREMENT WITH UNCERTAINTY ESTIMATES FOR A PELTON TURBINE GENERATING UNIT

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Abstract — The estimation of efficiency for a hydraulic electricity generating plant requires the measuring of the inlet water flow. This measurement is rarely available in most small and medium-sized hydraulic power plants, and was not available at the plant investigated. This paper discusses the solution developed to measure the inlet flow and therefore the efficiency of the generating unit. Confidence bands were estimated for the measurand (efficiency) based on Monte-Carlo simulations. The tuned algorithm has been tested using independent data collected at the hydroelectric power plant and agrees quite well with a validation set of measurements.

Keywords — Efficiency, turbine-generators, power plants instrumentation, power plant monitoring.

I. INTRODUCTION

The efficiency of hydraulic turbine-generators is of great practical importance. For instance, if a certain turbine-generator is known to have a low efficiency at certain load levels, operating at such points should be avoided in order to optimize water consumption, which is a growing concern nowadays. For instance, “for a small (i.e., 15 MW low-head unit, a 1% error in the definition of the best gate can result in a loss of tens of thousands of dollars in generation revenues per year for each unit” (Doering and Gawne, 1998). In recent works, turbine-generator efficiency has been used in defining a methodology for the optimal dispatch of generating units in a large hydroelectric plant (Arce et al., 2002) and in specifying a way of assessing economic performance and maintenance of hydraulic generating units (Liu et al., 2003). Important though as it is, very often electricity producing companies only have a rough estimate of the efficiency at nominal load.

In some cases, manufacturers provide a curve that shows how efficiency varies with the operating point (load). However, more often than not, such a curve is estimated based on reduced-scale models (in labs) or simply mathematically (Arce et al., 2002). For such reasons it is of practical interest to have a more realistic estimate of the efficiency of turbine-generators units in working conditions at power stations. Moreover, if such an estimate is available online, such information could be used in monitoring and fault detection schemes (Liu et al., 2003). It is believed that the last statement will be especially true if besides an estimate of efficiency an estimate of the its uncertainty is also available.

The practical estimation of efficiency in hydraulic turbine-generators is a task far from trivial. The main reason for this is that such an estimate strongly depends on plant variables, some of which are difficult to measure — such as the inlet water flow, or discharge (Doering and Gawne, 1998)— and some of which are obtained by data-fitted models — such as the pressure loss along the piping. This means that the estimated efficiency is uncertain. But how uncertain? This paper will describe the procedure followed to answer the above question in the context of a specific case study at a power station to be described briefly in section II. A key point in the procedure is the measurement of the inlet water flow, that will be described in section III. The procedure for calculating the efficiency is described in section IV and the Monte Carlo approach used to determine the uncertainty associated to the estimated efficiency is described briefly in section V. Finally, the main conclusions are given in section VI.

II. THE POWER STATION

The power station under consideration consists of a rather small natural water reservoir located over 360 m above the turbine-generator unit. The water flows from the reservoir to the turbine-generator through a long pipe with diameter equal to 1 m. The water volumetric flow is measured by means of a Cole (1935) type Pitot tube (more on this later) installed close to the reservoir— this results in relatively low absolute pressures— and far from any corners— this reduces
the influence of turbulence. In fact, the measuring plane is located over 200 m downstream the inlet at the reservoir and, on the other hand, the turbine is over 1200 m downstream the Pitot. This setting greatly favors flow measurement with a Pitot tube. The turbo generator is composed of a 7 MW Pelton turbine and a 6.8 MW Alstatom generator. Because the power station consists of this single turbine-generator with its single water inlet, these conditions are considered to be close to ideal for developing and testing a full procedure to determine the efficiency of turbine-generators based on pitometry.

III. INLET FLOW MEASUREMENT

Because of the importance of the inlet flow measurement, much work has been devoted to this issue in recent years. Therefore, before providing the details of the implemented method, a brief overview will be provided.

A. Background

Throughout the years a number of methods have been developed to estimate discharge (or inlet flow). Among such, some are widely accepted to be of difficult practical implementation, such as the Gibson pressure-time, tracer dilution and Allen salt velocity (Doering and Gawne, 1998). On the other hand, differential pressure, velocity-area and acoustical methods are considered to have several advantages over other techniques (Doering and Gawne, 1998; Liu et al., 2003). In particular, the differential pressure and acoustic or ultrasonic methods have been considered to be appropriate for real-time monitoring purposes (Liu et al., 2003).

The velocity-area method consists of using a number of propeller-type velocity sensors installed on a grid (Doering and Hans, 2001). Each sensor provides an estimate of the local velocity. The entire velocity profile has to be integrated in order to have an estimate of the flow. A crucial problem with this method is to determine the number and precise location of each sensor. In this respect, two standards DIN (1948) and ASME (1992) have been recently compared by Doering and Hans (1998) who concluded that the former overestimates the discharge by 3.09% on average whereas the ASME method underestimates it by 1.25% on average. Another important result of such a study is that the distribution of measuring points is far more important than their number for rectangular metering sections and for low-head turbines because in such situations the velocity profile is rather irregular and complex. The main disadvantages of the velocity-area method are (Doering and Gawne, 1998):

1. difficulty to correctly represent complex velocity profiles due to the finite number of measuring points;
2. the periodic calibration required for each propeller;
3. the difficulty of accurately measuring flow reversals;
4. the disruption of the flow field by the transversing apparatus.

A very promising alternative to the velocity-area method is the acoustical technique, which consists of measuring the average velocity of the flow by means of ultrasonic transducers (piezoelectric crystals). This is achieved by measuring the transit-time of an ultrasonic beam. The result is a chordal average velocity at each measuring point along the metered area. The set of average velocities still needs to be integrated numerically in order to yield the total flow (Doering and Gawne, 1999). In order to collect such velocities a set of acoustical transducers has to be duly installed and, in order to guarantee high quality measurements, installation details are critical (Vos et al., 1996).

It should be realized that both, the velocity-area and acoustical methods require numerical integration. However, there is an important difference which is that whereas the velocity-area requires integration in both directions of the metering grid, the acoustical method requires integration only along one direction. The other direction is automatically integrated by the averaging procedure underlying the method (because the ultrasonic beam crosses the velocity field in one direction).

The method to be described in the following section is, in some respects, similar to the velocity-area method. However some of the main disadvantages mentioned above (see items 1–4) have been, at least partially, overcome as discussed below.

Finite number of measuring points. In fact, the number of measuring points in the implemented method is finite. However, because of installation conditions (see Sec. II), the velocity profile can be considered fully developed and symmetrical. This means that with a relatively small number of measuring points an approximate overall velocity profile can be obtained.

Periodic calibration of each propeller. In the procedure implemented the velocity is measured by a Pitot tube. One of the benefits of this is that because such a device has no moving parts the time interval between calibrations can be greatly increased.

Flow reversals. In fact the method used does not account for flow reversals, but this situation never happens in downstream flow measurement in the power plant under study.

Disruption of the flow field. Because only one Pitot tube is used (which is inserted in the piping during normal operation) the disturbance on the flow field is greatly reduced. The price to be paid for this is a painstaking procedure of mapping the flow field by positioning the Pitot tube successively at each measuring point (see Kang et al., 2000) for an interesting
illustration of this fact in the context of gas flow measurement), for each load level. This has been done in loco and takes several days and requires the possibility of setting the power plant at various operating points. Fortunately, due to its size and load demands the power plant under study fulfills all such requirements.

B. The Implemented Method

Because of a number of practical issues, in this work it was decided to measure the volumetric flow at the power station using pitometry. A Pitot tube is a device for which the differential pressure between its taps and the flow velocity are related by (Doebelin, 2003):

\[
V = \sqrt{\frac{2(P_{\text{stag}} - P_{\text{stat}})}{\rho}},
\]

where \( V \) is the velocity of the flow at the Pitot location, \( P_{\text{stag}} \) is the stagnation pressure, \( P_{\text{stat}} \) is the static pressure and \( \rho \) is the fluid density. Assuming the Pitot tube is placed at the center of the piping, the flow is given by

\[
Q = F_v V_c S_{\text{ef}} C_1 C_4,
\]

where \( Q \) is the volumetric flow in m\(^3\)/s, \( F_v \) is the velocity factor, \( V_c \) is the average velocity (m/s) at the center, \( S_{\text{ef}} \) is the effective area in m\(^2\). \( C_1 \) is an area correction coefficient due to the presence of the Pitot inside the piping and \( C_4 \) is the diameter correction coefficient that takes into account the difference between the nominal and real diameter.

In this work \( S_{\text{ef}} \) was considered constant and equal to \( S_{\text{ef}} = 0.7854 \) (Andrade, 2002). The tap used to introduce the Pitot tube into the piping does not interfere with the flow and therefore \( C_1 = 1 \). Likewise, because the pipe wall is very thin compared to the total diameter (1 m), in the present application, \( C_4 = 1 \) was chosen. Finally, the velocity factor is given by

\[
F_v = \sum_{i=1}^{10} \frac{\sqrt{\Delta P_i}}{10^{\sqrt{\Delta P_c}}},
\]

where \( \Delta P_i \) (\( \Delta P = P_{\text{stag}} - P_{\text{stat}} \), see (1)) are the differential pressures at the various central radii and \( \Delta P_c \) is the differential pressure at the Pitot taps when it is placed at the center of the piping. The velocity factor quantifies the flow velocity profile. A totally flat profile would yield \( F_v = 1 \). \( F_v \) decreases as the profile becomes sharper. Besides, the shape of the velocity profile does change with the flow itself. Thus, the velocity profile was measured at the power station at various operating points and the following exponential function was fitted

\[
F_v(\Delta P_c) = 1.0349 \Delta P_c^{-0.0339}.
\]

It is important to notice that the numerical integration required in flow measuring methods based on estimates of local velocities is implied in Eq. (2) and Eq. (3). In Eq. (3) it is clear that each measuring point receives an equal weight in the integration. In fact, the method used determines the location of the measuring points in such a way as to define imaginary cylinders of equal volume (Vennard, 1963). The investigation of other ways to carry out the numerical integration, especially when the velocity profile cannot be considered symmetrical, is certainly an important field of research (Doering and Gawne, 1999).

In this work the measuring locations are at 25, 81, 146, 226, 342 and 500 mm from the pipe wall. Since the diameter is 1 m, the last measuring point is located at the center of the tube. These locations assume symmetry, which in this case study was considered a realistic assumption (see Sec. II). The local velocity at each of the aforementioned locations was measured at various load levels over six tests (for each load). Average values are show in Fig. 1 and corresponding standard deviations are shown in Fig. 2.

C. Calibration

It is known that in practice Eq. (1) need to be corrected by a factor that will depend on the shape and position of the Pitot tube orifices. Although such a factor is usually provided by the manufacturer, it can vary significantly between 0.86 and 1 (Vennard, 1963). In view of this, it was decided to calibrate the Pitot tube using a LASER-Doppler anemometer (LDA) and to find the precise correction factor required for this particular device. The calibration data were used to arrive at the following corrected expression (Andrade,

![Graphs showing standard deviation of velocity](image)

Figure 2: (a) Standard deviation (among different tests at the same load and position) of the measured velocity. (b) Same as in (a) but with values in percentage of the central velocity (see Fig. 1). At the center, for all loads, the standard deviation is around 1.5%. (●) 1 MW; (—) 2 MW; (— —) 3 MW; (— — —) 4 MW; (— — — —) 6 MW.

Figure 3: Differential pressure transmitter as installed at the power plant.

(2002)

\[ V_e = 0.0937\Delta p_c^{0.51}. \]  \hfill (5)

In order to use Eq. (5) to determine the central velocity and subsequently to obtain the water flow \( Q \), it is necessary to measure the differential pressure \( \Delta p \) produced by the Pitot tube. To this end, an electronic device based on a piezoresistive pressure sensor has been designed, built, tested and installed, see Fig. 3. This device includes a temperature sensor (a Pt100 RTD) whose output is used to compensate for environmental temperature fluctuations. Calibration both of temperature and pressure was performed in the laboratory, and spectral analysis of the data collected at the power station over several days confirmed that temperature compensation was successful.

Finally, given a measurement of \( \Delta p_c \); obtained with the Pitot in online operation; the velocity factor in Eq. (4) and the central velocity in Eq. (5), the volumetric flow in Eq. (2) can be obtained. Such a measurement plays a key-role in the calculation of the turbine-generator efficiency, as it will be discussed in the next section.

It is well known that any flow measurement technique, especially those based on differential pressure measurements, become rather unreliable for low values of the flow, in particular in the first 1/3 of the scale (Doebelin, 2003). In fact, in the situation investigated in this work, at load levels less than 2.5 MW the measured \( \Delta p_c \) is less than 10% of its value at full load, 6.8 MW. Therefore, at such low load levels a computed flow value, rather than the measured one, is used to determine the efficiency, as explained below.

For load values under 2 MW the electrical signal corresponding to the Pitot pressure drop exhibits fluctuations that eventually reach negative values in normal operation during transients. Clearly this does not reflect the real situation. In order to handle these cases, one could arbitrarily set the differential pressure value to a small nonzero constant. However, this would lead to non-smooth volumetric flow data. The adopted approach was to employ the following convex combination:

\[ \Delta p_c = x\lambda + \theta_1e^{\theta_2(x-\theta_1)}(1-\lambda); \]  \hfill (6)

\[ \lambda(x) = \frac{1}{1+e^{-\theta_2(x-\theta_1)}}; \]

where \( x \) is the differential pressure actually measured from the Pitot tube; and \( \theta_i, i = 1, \ldots, 4 \), are parameters conveniently adjusted to ensure, for load values from 2 MW to 3 MW, a smooth transition between the calculated and the measured value of volumetric flow.

For load levels 3 MW and higher a reliable measurement of the flow is attained and used in the calculation of the efficiency. It should be realized that because the turbine-generator efficiency is known to be poor at low load levels, operating at such points should be avoided in normal operating conditions and therefore the approximation just described has no impact whatsoever on the final estimated efficiency from a normal operation point of view.

Finally, in order to illustrate the on-line ability of the system to measure differential pressure, Fig. 4 shows a step change in load, from 5 MW to 6 MW. Not only \( \Delta p_c \) increases, but also its variance. This is due to the increase of turbulence with load.

**IV. DETERMINING EFFICIENCY**

The electric power produced by the generator is measured at the power station. Therefore, in order to determine the efficiency, it is necessary to estimate the amount of hydraulic power actually delivered to the turbine, which strongly depends on the inlet flow.

The net power, \( P_n \), equals the total power \( P_t \) minus the pressure loss along the piping \( \Delta h_f \). Using a gauge
V. ESTIMATING UNCERTAINTIES

The procedure for measuring the inlet flow, described in section III and the estimation of the efficiency itself (see section IV), employ a number of measurements and coefficients which are determined amidst uncertainty. Obviously, the uncertainty associated to each coefficient and measured value used in the aforementioned procedures have a final effect on the estimated efficiency $\eta$. The analytical estimation of uncertainty of measurement uses the propagation law to determine how individual uncertainties in the independent variables propagate to the final result. Such a procedure typically involves calculating partial derivatives (Doebelin, 2003; IEC-ISO, 2004) and some of its practical and conceptual difficulties have been pointed recently (Golubev, 2008). Unfortunately, given the complex way in which the many variables involved relate to each other in the present context, it is not feasible to analytically find an expression that will give the combined standard uncertainty of measurement as a function of the standard uncertainty of each independent measurement (Locci et al., 2002).

Therefore, in order to determine the combined standard uncertainty on $\eta$, a numerical procedure based on Monte Carlo simulation was followed. A number of issues related to the numerical determination of uncertainty has been recently discussed in IEC-ISO (2004) and Angrisani et al. (2006), and, in particular, a Monte Carlo approach to uncertainty determination was discussed in Locci et al. (2002).

A probability density function (PDF) was defined for the differential pressure at the Pitot taps $\Delta p_c$. Subsequently, a large number of realizations were taken (2500 for the results shown in this paper) from that PDF and for each realization the full procedure for determining the efficiency was followed. This was done over the entire operating range, that in terms of the differential pressure at the Pitot taps goes from 1 to 1300 mm H$_2$O. The Pitot full scale 1300 mm H$_2$O corresponds to 6.8 MW. So finally, instead of a single value for each variable an ensemble of values was obtained for which the respective mean and standard deviation can be readily computed.

As seen in Fig. 6, the distribution of actually measured $\Delta p_c$ is quite Gaussian, especially at lower load levels. The increase in the dispersion of $\Delta p_c$ with load is due to turbulence and not due to the increase in uncertainty of measurement of this variable. Based
on such measurements, the probability distribution in the corresponding Monte Carlo simulation was taken to be Gaussian. As a consequence only the first two moments, mean and variance, need to be chosen to completely specify it. Calibration with the LASER-Doppler anemometer resulted in a standard uncertainty for the Pitot differential pressure measurement of 2%FS (this corresponds to 24 mmH₂O). Therefore, a standard deviation of 2% was used for the Gaussian PDF of variable Δpᵣ. Due to calibration, it was assumed that the PDF of Δpᵣ was unbiased, that is, the mathematical expectation of such a random variable coincides with the true value of this quantity, which means that there would be zero systematic measurement error in a metrological context. These assumptions seem to be supported by the measurements at 0 MW (see Fig. 6). At this load, water flow is not completely zero.

The full procedure can be summarized as follows:

1. set an initial value for Δpᵣ;
2. take N realizations from a Gaussian distribution with mean Δpᵣ and standard deviation σΔp = 24 mmH₂O which is 2% of 1200 mmH₂O (full scale). In this paper N = 2500;
3. using Eq. (5) and the N values of Δpᵣ, compute N values of Vᵣ;
4. using Eq. (4) and the N values of Δpᵣ, compute N values of Fᵣ;
5. using Eq. (2), the N values of Vᵣ and Fᵣ, and the constants S RoundedRectangle, Cᵣ = 1 and C₃ = 1, compute N values of Qᵣ;
6. using Eq. (7) and the N values of Vᵣ, compute N values of Δhᵣ;
7. using Eq. (9), N values of Q and Δhᵣ, and taking Hᵣ = 362.41 mH₂O, compute N values of the net power delivered to the turbine Pₑ;
8. using Eq. (11) and N values of Δpᵣ, compute N values of Pₑ;
9. using Eq. (10) and N values of Pₑ and Pₑ, compute N values of nₑ;
10. increment the value of Δpᵣ, and return to the second step until Δpᵣ = 1200 mmH₂O (full scale) is reached.

It should be noticed that in real-time operation, the generated electrical power is measured rather than computed using Eq. (11).

At full scale (6.8 MW of generated power and 1200 mmH₂O of differential pressure at the Pitot taps) the standard deviations of the resulting propagated PDFs through the measurement model are (see Fig. 7)

- \( σ_{Δp} = 24.16 \text{ mmH}_2\text{O}, \) (2%)
- \( σ_{Fᵣ} = 0.0071, \) (0.71%)
- \( σ_{Vᵣ} = 0.062 \text{ m/s}, \) (1.5%)
- \( σ_{Δhᵣ} = 1.189 \text{ m}, \) (5.7%)
- \( σ_{Q} = 0.043 \text{ m}^3/\text{s}, \) (1.9%)
- \( σ_{Pₑ} = 0.140 \text{ MW}, \) (2.1%)
- \( σ_{Pₑ} = 0.263 \text{ MW}, \) (3.9%)
- \( σ_{nₑ} = 3.82\%. \)
It should be realized that only $\sigma_{\Delta \rho}$ was input to the algorithm and that all the remaining standard deviations are the outcome of the Monte Carlo simulations.

The Monte Carlo simulation was tuned varying the order of the polynomials fitted by linear regression techniques. An important detail is that before model fitting, the data were log-transformed. Therefore the polynomial models acting on log-transformed data work as exponential models. This procedure enables the use of linear regression techniques to fit exponential models. After several trials, it became clear that the best order for the log-transformed polynomial models was one. When this value was increased, the final efficiency curve revealed fluctuations which, from experience, are known to be spurious. The outcome of this tuning stage was an estimate of the measured efficiency as the corresponding expected value associated to the PDF for each operating point considered, together with estimated standard uncertainty of measurement taken as the standard deviation from each PDF. This outcome is shown in Fig. 8, where a confidence band is depicted considering an expanded uncertainty with unity coverage factor ($k = 1$) (IEC-ISO, 2004).

After tuning was completed, a totally different set of data was collected at the power station over fifty days. The various online measurements, including $Q$, were recorded. Such data were fed to the efficiency calculation procedure. The calculated values for each measured situation are shown with crosses in Fig. 8 together with the outcome of the Monte Carlo simulation. The agreement is good overall and therefore the tuned Monte Carlo simulation was considered valid. It is also worth noticing that for the chosen coverage factor ($k = 1$) the corresponding level of confidence is only 68.27% (Gaussian distribution) that the true measurement can be found inside the confidence bands.

In this context, some disagreement can be seen at 1.5 MW, and to a lesser extent at 3 MW, and at nominal load 6.8 MW. The disagreement at 1.5 MW is only partial since most of the measured efficiency values fall inside the $\pm 2\sigma$ bands. At 3 MW all the computed values are outside the confidence bands. The authors are convinced that this is due to the fact that this value of power is precisely the threshold value for the second turbine needle to start operating. This introduces a discontinuity in the efficiency calculation that has not been taken into account in the simulations.

Finally, a very slight disagreement is seen at full scale load, which seems to suggest that the actual efficiency curve is somewhat flatter than what has been estimated. It is reminded that the efficiency curve was calculated based on measurements only up to 6 MW. From this value up to 6.8 MW, the efficiency mean and confidence bands were simply extrapolated. This could also account for some of the mismatch at full scale.

Figure 8: The middle line is the average calculated efficiency and the sidebands are plus and minus one standard deviation at each generated power as computed by the tuned Monte Carlo simulation. The exponential models used in the procedure—see Eqs. (4), (5), (7) and (11)—were fitted to data measured up to 6 MW. Therefore, from that load the thinner lines indicate that they are extrapolations beyond the data limits. The crosses indicate efficiency values computed from data measured at the power station for the sake of validation. Agreement is generally good.

In closing this discussion, it is interesting to notice that in a recent paper generator efficiency was only computed from 60% to full load (Arce et al., 2002) as it would be expected, especially for very large turbine-generator units (700MW). In that same paper, it was reported that the overall turbine-generator efficiency varied in the range 67% < $\eta$ < 94%. In Fig. 8 it can be seen that the mean efficiency varies in the range 80% < $\eta$ < 95% and, considering the confidence bands, such a range is extended to 65% < $\eta$ < 100%.

VI. CONCLUSIONS

This paper has described the procedure for determining the efficiency of a 7MW turbine-generator installed at a hydroelectric power plant. Standard procedures for calculating efficiency do not usually provide any uncertainty evaluation. In this paper, a power efficiency measurement technique was presented, together with a Monte-Carlo-based procedure used to estimate the associated standard uncertainty of measurement. Therefore confidence bands for the power efficiency obtained from measured data could be provided.

The technique relies on differential pressure measurements from a Pitot Tube installed downstream the water inlet at the reservoir of the hydroelectric power plant. Extensive online measurements revealed that the differential pressure is quite Gaussian, especially at low load levels.

The free parameters used to tune the Monte Carlo simulation was the model orders used to fit the data.
Such models yield important variables such as the velocity factor and the pressure loss in the inlet piping. After tuned, the final result was validated with data measured at the power station over fifty days of normal operation. It was found that the tuned algorithm did agree very well with the measured data.

A very important byproduct of the Monte Carlo procedure is the possibility to assess the sensitivity of the proposed efficiency estimation procedure to variations in each independent variable. As expected, the inlet water flow is a very influential variable. Also, the velocity factor is quite critical. This shows that the described procedure will probably require further improvements (e.g. installation of a second Pitot tube or use of acoustic techniques) in cases where the flow profile is more irregular or less symmetrical.

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