AN ANALYTICAL STUDY OF RADIATION EFFECT ON THE IGNITION OF MAGNESIUM PARTICLES USING PERTURBATION THEORY

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Abstract— An analytical method is proposed to solve the heat Energy equation of combustion of magnesium particles with allowance for the heterogeneous chemical reactions and the region of the thermal influence of the particle on the gas. In this model the solution of the problem in a steady formulation is found and the flame propagation mechanism is considered to be radiation and conduction. Radiative heat transfer plays a major role in single particle combustion. Flame equations of the single particle combustion are solved via the new nonlinear differential equation by using perturbation theorem.

In this paper we can apportion the magnesium-particle ignition to regular regimes and also regime of particle extinction and the ignition. Under the steady approximation, the values of particle and gas phase temperatures are calculated and by adding radiation term in conservative equations, both of these parameters will be increased.

Keywords — heterogeneous, Radiative heat transfer, nonlinear equation, perturbation theorem.

I. INTRODUCTION

The problem of physicomathematical modeling of ignition and combustion of metal samples is of considerable interest for various branches of industry (Fedorov et al., 2003).

The main objects described in Fedorov et al. (2003) are the pointwise and partly distributed models of ignition of small metal particles with low-temperature oxidation proceeding on the particle surface. The heat dissipated in to the gas phase was ignored. This means that the thickness of the so called surrounding film is negligibly small. It seems of interest to study the effect of this parameter on the thermal history of the reacting particle diameter plays a significant role in determining the relevant combustion mechanisms by affecting the characteristic transport diffusion time relative to the chemical kinetics time. A large diameter particle at high pressure may burn under diffusion controlled conditions, whereas a small particle at low pressure may burn under kinetically controlled conditions (Yetter and Dryer, 2001; Huang et al., 2006). The oxidizer type also has strong effects on magnesium particle combustion, since the flame and surface temperatures can be affected by transport in the surrounding gas. In the last two decades with the rapid development of nonlinear science, an ever increasing interest of scientists and engineers in the analytical techniques for nonlinear problems appeared. The widely applied techniques are perturbation methods. Perturbation method is one of the well-known methods to solve the nonlinear equations which was studied by a large number of researchers such as Bellman (1964), Cole (1968) and O’Malley (1974). A valuable model of ignition of single magnesium particle, proposed by Fedorov and Shul’gin (2006), is expanded in this study. In previous theoretical models (for example, Fedorov et al., 2003; Fedorov and Shul’gin, 2006; Fedorov, 1994; Fedorov, 1996) it is assumed that radiative heat transfer was neglected. The radiation term is added to the previous model (Fedorov and Shul’gin, 2006), and its effect on the combustion of magnesium particle is investigated. The initial investigation of the combustion of bimodal aluminum and iron particles is also studied with help of the presented model.

II. MATHEMATICAL FORMULATION

Combustion of fine metal particles, which takes place in the gas phase (where evaporated sample particle reacts with oxidizer) is formulated here. We consider a metal particle of radius $r_p$ surrounded by a gas layer of thickness $L-r_p$. We assume an exothermic chemical reaction of oxidation to proceed on the sample surface.

The main focus of this research is made on the effect of radiation on the combustion of metal dust particles. This paper develops the previous model published by Fedorov and Shu’l’gin (2006).

Then the mathematical model that describes the temperature fields in general case $T$, has the form

$$\rho_c v_i \frac{\partial T_i}{\partial t} = \lambda_i \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_i}{\partial r} \right) + Q_{rad}.$$  \hspace{1cm}(1)

The temperature of the ambient gas and sample particle are indicated by $T_g$.

In the previous relationship, $v$ is the factor of symmetry equal to 0, 1, and 2 for the planar, cylindrical, and spherical cases, respectively, $\rho_c$, $\lambda_i$, and $c_i$ are the density, thermal conductivity, and specific heat of the phase; the subscript $i=1$ and 2 refers to parameters of the gas and the particle respectively. $Q_{rad}$ is the radiation heat transfer, which was not taken into account in previous models but is considered in the present article. In fact, it distinguishes the present study from the previous works in this field.

Radiation heat transfer is given by the Stefan-Boltzmann law,

$$Q_{rad} = \sigma T^4,$$

where $\sigma$ is the Stefan-Boltzmann constant.
\[ Q_{\text{rad}} = \varepsilon \sigma (T^4 - \bar{T}^4). \quad (2) \]

where \( \sigma \) and \( \varepsilon \) are the Stefan-Boltzmann constant and emissivity of the flame, respectively. In addition \( \bar{T} \) is the temperature at the boundary of the gas region.

In the present study, the steady state problem is investigated and therefore the governing equations for gas and particle are represented as follows:

\[
\lambda_g \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T_g}{\partial r} \right) + \varepsilon_g \sigma (T_g^4 - \bar{T}^4) = 0 \quad 0 \leq r \leq r_p
\]

and

\[
\lambda_p \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T_p}{\partial r} \right) + \varepsilon_p \sigma (T_p^4 - \bar{T}^4) = 0 \quad r_p \leq r \leq L
\]

The three needed boundary conditions for Eqs. (3) and (4) are:

\[ r = 0 : \frac{\partial T_g}{\partial r} = 0 \]
\[ r = r_p : T_g = T_p \]
\[ r = L : \frac{\partial T_p}{\partial r} = -\alpha (T_g - \bar{T}) \]

where \( q_0 \) is the heat release per unit mass of the oxide, \( \rho_v \) is the oxide density, \( h \) is the oxide-film thickness, \( dh/dr \) is the rate of variation of the oxide-film thickness.

Moreover, \( K \) represents the frequency factor, \( E \) the activation energy of low-temperature oxidation. To be the universal gas constant, the heat-transfer coefficient is calculated as follows (Fedorov et al., 2003),

\[ \alpha = \frac{\lambda \nu}{2 \rho \nu r_s} \]

After some simplifications, this coefficient is calculated by \( \alpha = Nu/2 \), where the Nusselt number is taken as a constant equal to 2 (Fedorov and Shul’gin, 2006). In this research, the radiation effect is considered from the gas and particle to the surrounding environment with the temperature of \( \bar{T} \). Since the ambient temperature is lower than the gas and particle temperatures, the radiative heat transfer between the oxidizer and particle is neglected compared to the radiative heat transfer from the gas and particle to the surrounding environment.

III. DIMENSIONLESS FORM OF EQUATIONS

We can write particle and gas phase energy conservation Eq. 3 and Eq. 4 in a non-dimensional form by using the following non-dimensional parameters:

\[
\tau = r \rho_p, \quad \xi = L \rho_p, \quad T_g = T_p \rho \nu r_s, \quad \sigma = \frac{q_0 \lambda}{\rho \nu r_s}, \quad E = \frac{E}{RT_g}, \quad \bar{T} = \frac{\bar{T}^4}{\bar{T}_0^4} \]

Since the energy equation in each system (i.e., gas and particle) is conserved, the total energy of this system (gas+particles) in the preheated zone of the gas-particles mixture is conserved.

As stated in Fedorov and Shul’gin (2006) the parameters have the following values:

\[
\lambda_g = 1.29 \frac{m_k}{m}, \quad \lambda_p = 174 \frac{m_k}{m}, \quad \rho_v = 3600 \frac{m_k}{m}, \quad \rho_i = 1000 \frac{J}{kg K}, \quad c_1 = 11000 \frac{J}{kg K}, \quad \rho = 2.471 \times 10^3 \frac{W}{m k}, \quad \lambda_p = 156 \frac{W}{m k}, \quad E = 4.31 \times 10^5 \frac{J}{kg}, \quad R = 284 \frac{J}{kg K}, \quad T_0 = 300 K
\]

The value of these parameters for the other particles (Al, Fe) obtained from references.

By substituting dimensionless parameters in the governing equations, we can write the equations obtained for each phase.

\[ P \alpha = -\varepsilon \sigma (T_g^4 - T_p^4) \]
\[ G \alpha = -\varepsilon \sigma (T_g^4 - \bar{T}^4) \]

The values of the non-dimensional emissivity (\( \varepsilon \)) for particle and gas are defined as:

\[ \varepsilon_g = \frac{\varepsilon_g \sigma T_g^4}{\lambda_p} \]

\[ \varepsilon_p = \frac{\varepsilon_p \sigma T_p^4}{\lambda_p} \]

IV. ANALYSIS OF THE RADIATIVE HEAT TRANSFER PROBLEM USING THE PERTURBATION TECHNIQUE

Exact solutions are rare in many branches of fluid mechanics, solid mechanics and other physical phenomena because of nonlinearities, non-homogeneities, etc. Hence in engineering and physics we are forced to determine approximate solutions of the problem they are facing. These approximations may be purely numerical, purely analytical, or combination of numerical techniques.

In this work, we use the perturbation theory to determine approximate solutions of differential equations that govern the phenomenon. Perturbation techniques (Nayfeh, 1973; Naives, 1979) are widely applied for obtaining approximate solutions to these equations involving a small parameter \( \varepsilon \). According to these techniques, the solution is represented by the first few terms of an asymptotic expansion, usually not more than two terms. The expansions may be carried out in terms of a parameter (small or large) which appears naturally in the equations, or which may be artificially introduced for convenience. In this article, the system of Eq. 3 and Eq. 4 for the particle and the gas is nonlinear and heterogeneous. By using perturbation series we need to expand our differential equation in power series of a small parameter \( \varepsilon \),

\[ T_i = \sum_{n=0}^{\infty} \varepsilon^n T_i^n = T_i^0 + T_i^1 \varepsilon + T_i^2 \varepsilon^2 + O(\varepsilon^3) \]  

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Considering the accuracy that is needed for this work, the final solution of Eq. 3 and Eq. 4 is considered up to first order

**Zero order**

**Particle phase:**

\[
\begin{bmatrix}
L_r \frac{\partial}{\partial \tau} \left[ T_0 \frac{\partial T_{0_1}}{\partial \tau} \right] \right|_{\tau=0} = 0, \quad 0 \leq \tau \leq 1
\]

**Gas phase:**

\[
\begin{bmatrix}
L_r \frac{\partial}{\partial \tau} \left[ T_0 \frac{\partial T_{0_1}}{\partial \tau} \right] \right|_{\tau=1} = 0, \quad 1 \leq \tau \leq L
\]

The boundary conditions for Eqs. (12) and (13) are obtained from the boundary condition (5). The boundary condition (14) for zero-order equation is determined by the perturbation theory. The leading term of two phase temperatures \( T_{0_1} \) satisfies the boundary conditions

\[
\begin{align*}
\tau = 0 & \quad \frac{\partial T_{0_1}}{\partial \tau} = 0 \\
\tau = 1 & \quad T_{0_2} = T_{0_1} \\
\tau = L & \quad \frac{\partial T_{0_1}}{\partial \tau} = -\alpha \left( T_{0_1} - \bar{T} \right)
\end{align*}
\]

The physical meaning of these boundary conditions is described as below:

- The symmetric condition is assumed at \( \tau = 0 \).
- Two boundary conditions are needed at \( \tau = 1 \). The first one is the matching condition for the temperature profile and the next one is the equality of heat flow from the particle and heat flow from the ignition of the particles.
- At \( \tau = L \), the external heat flow from the oxidizer is equal to the convective heat transfer from the surrounding gas. According to Fedorov et al. (1996) and based on the present radiative heat transfer, the heat release due to the oxidation can be neglected with a good approximation.

**First – order**

\[
\begin{align*}
\begin{bmatrix}
L_r \frac{\partial}{\partial \tau} \left[ T_0 \frac{\partial T_{0_1}}{\partial \tau} \right] \\
L_r \frac{\partial}{\partial \tau} \left[ T_0 \frac{\partial T_{0_1}}{\partial \tau} \right]
\end{bmatrix} & = \bar{T}^4 - T_{0_1}^4, \quad 0 \leq \tau \leq 1
\end{align*}
\]

The boundary conditions for Eq. 15 and Eq.16 are as follows:

\[
\begin{align*}
\tau = 0 & \quad \frac{\partial T_{0_1}}{\partial \tau} = 0 \\
\tau = 1 & \quad T_{0_2} = T_{0_1} \\
\tau = L & \quad \frac{\partial T_{0_1}}{\partial \tau} = -\alpha \left( T_{0_1} - \bar{T} \right)
\end{align*}
\]

Since the final solution is continuous for both particle and oxidizer and the ignition process is assumed homogeneous, the temperature equilibrium of gas and particles seems to be a logical assumption.

Since it is considered that the surrounding gas is heated by the output source (such as electrical power), the surrounding gas temperature remains constant. It is a reasonable assumption which is supposed by lots of researchers in the experimental apparatus, so \( \bar{T} = \text{cte} \).

Under the steady approximation, the heat transfer equation in each phase is solved by matching the heat fluxes on the boundary of solid particle phase and gas phase; and the temperature distribution is consequently obtained.

The equations that describe the zero order of temperatures of the solid particle and the gas have solutions of the form

\[
T_{0_1} = -\frac{a_1}{(v-1)\bar{T}^{v-1}} + b_1
\]

where \( v = 0, 2 \) and \( a_1 \) and \( b_1 \) \((i = 1, 2)\) are constants. Note that the case of cylindrical symmetry is considered in a similar manner; a logarithmic function is used as the basic solution. It follows from the condition at the point of symmetry that \( T_{0_2} = \text{cte} \). Using the conjugation of temperatures and heat inflow at the interface between the particle and the gas (Eq. 5), and also the condition of heat transfer at the outer boundary and after some transformations, we find a function that describes the catastrophe/ignition manifold in the form

\[
\beta = \beta(\alpha, L, \chi, v, E, \bar{T}) = \frac{\exp\left(-\frac{E}{T_{0_1} - \bar{T}}\right)}{T_{0_1} - \bar{T}} = \frac{\exp\left(-\frac{E}{T_{0_1} - \bar{T}}\right)}{T_{0_1} - \bar{T}}\frac{\alpha T^\nu}{\nu - 1} \frac{T_{0_1} - \bar{T}}{\alpha T^\nu}
\]

This expression determines the leading term of particle temperature; and after substituting the defined temperature into the related boundary conditions we find the coefficients \( a_1 \) and \( b_1 \), i.e., the gas temperature in the region surrounding the particle, in the form

\[
a_1 = -\frac{\alpha T^\nu}{1 - \alpha T^\nu/v} + b_1
\]
\[ b_1 = T_0 \frac{\pi \tau^{1/2} \left( \frac{\beta}{\tau} - \tilde{T} \right)}{\left( 1 - \tau \right)^{1/2}} \quad \nu \neq 1 \]

Using the methods of the elementary catastrophe theory, we can obtain the turning points of the ignition manifold from the equation \( \frac{T_0^2}{\nu} - E \tilde{T}_{0,2} + 2 \tilde{T} = 0 \), which has two roots:

\[ \tilde{T}_{0,2} = \frac{E}{2} \left( 1 \pm \sqrt{1 - \frac{4E}{\nu}} \right) \]

Then, in accordance with (Fedorov, 1994; Fedorov, 1996), such a manifold can be constructed in the plane \( \{ \beta, \tilde{T}_{0,2} \} \), which is schematically shown in (Fedorov and Shul’gin, 2006), where \( \beta_{\pm} \) are the values calculated at the turning points, i.e., for \( \tilde{T}_{0,2} = \tilde{T}_{0,2,\pm} \).

Rigorously speaking, it is seen from (Fedorov and Shul’gin, 2006) that the steady solution is not unique. On the lower branch \( I \) with \( \beta \geq \beta_{\pm} \), the solution \( \tilde{T}_{r} \in \{ \tilde{T}_{0,2,\pm}, \tilde{T} \} \) describes the regime of regular heating.

For \( \beta \geq \beta \geq \beta_{\pm} \), we have \( \tilde{T}_{r} \in \{ \tilde{T}_{0,2,\pm}, \tilde{T} \} \), it should be noticed that branch II is unstable (Fedorov and Shul’gin, 2006). Finally, for \( 0 \leq \beta \leq \beta_{\pm} \) we have \( \tilde{T}_{r} \geq \tilde{T}_{0,2,\pm} \), and this solution belongs to branch III, which corresponds to the phenomenological criterion of ignition. Based on these facts, we can formulate the following statement. The quantity \( \beta \), which defines the dynamics of particle heating (regular or with ignition) is a decreasing function tending to a certain constant as \( \beta \to \infty \).

This is in agreement with an actually observed fact that injection of an additional mass of the gas into the system increases the ignition limit of the sample.

This assumption is illustrated in Fig. 1 and Fig. 2, shows the dependency of \( \beta_{0,0} \) on \( \beta \) for \( \text{Nu} = 2 \) for the spherical and planar cases respectively. It can also be noted that the ignition limit is higher in the spherical case, because the criterion of regular development of the process \( 0 \leq \beta \leq \beta_{\pm} \) can be more readily satisfied. Indeed, \( \beta(\nu = 0) < \beta(\nu = 2) \).

\[ \text{Figure 1. Dependency of } \beta_{0,0} \text{ on } \beta \text{ for spherical case} \]

\[ \text{Figure 2. Dependency of } \beta_{0,0} \text{ on } \beta \text{ for planar case} \]

The next important step is the computation of the first order temperature of particle and gas phase. By superposing the leading term of two phase temperatures into Eq. 15 and Eq. 16 the system of differential equations in each phase can be solved.

By solving the system of differential equations in each phase, the first order of particle and gas temperature in the cases of planar and spherical coordinates can be found.

**Particle Temperature**

\[ \tilde{T}_{1} = \frac{A \tau^{2}}{2(1+\nu)} + C_{1}, \quad (20) \]

where

\[ A = \frac{1}{\pi} \left( \tilde{T}^{4} - \tilde{T}_{s}^{4} \right). \]

**Gas Temperature**

**Planar coordinate**

\[ \tau_{1} = \frac{1}{2 \pi} \left( -a_{1}^{4} \tilde{T}^{2} - a_{1}^{2} \tilde{T}_{1}^{2} - \frac{4a_{1}^{4} b_{1}^{3} \tau^{3} - 4a_{1}^{4} b_{1}^{3} \tau^{3}}{10} - \frac{a_{1}^{4} \tilde{b}_{1}^{3}}{15} \right) \]

**Spherical coordinate**

\[ \tau_{1} = \frac{1}{2 \pi} \left( -a_{1}^{4} \tilde{T}^{2} - 2a_{1}^{4} b_{1}^{3} \tau^{2} - \frac{4a_{1}^{4} b_{1}^{3} \tau^{2} - 4a_{1}^{4} b_{1}^{3} \tau^{2}}{6} + \frac{\tilde{T}^{4} \tau^{2} - \tilde{T}^{4} \tau^{2}}{6} \right) \]

From the procedure of matching boundary conditions obtained from solutions on both sides of the boundary (\( \tilde{T} = 1 \)) and also from system of boundary conditions (17) at \( \tilde{T} = 0 \) and \( \tilde{T} = \tilde{T} \), the parameters \( C_{2}, C_{3}, C_{4}, C_{5}, C_{6} \) for Eq. 20 and Eq. 21 are found.

**V. RESULTS AND DISCUSSION**

In order to find the influence of the oxide-film thickness on the ignition of magnesium particle, some computations with different sizes of \( \tilde{T} \) were performed.

The results plotted in Fig. 3 show that the main changes in temperature of the system occur mainly near...
the contact boundary, with $r < 4$. In this region, the temperature gradients acquire the highest values. If the region occupied by the gas is large, the temperature changes only weakly. This allows us to argue that the boundary of the region occupied by the gas can be assumed to be approximately equal to four particle radii.

Increasing the values of gas thickness ($L$) gives rise to the gas temperature gradient; This phenomenon is understood considering a layer of insulation which encloses the spherical solid particle. The inner temperature of the insulation is the particle phase temperature $T_2$, and the outer surface is exposed to a convection environment (boundary of the gas region) at $T_1$.

It can be proven that the “critical radius of insulation” for the spherical particle is found as follows:

$$r_c = \frac{2\lambda}{\alpha}.$$

If the outer radius is less than the value given by this equation, then the heat transfer will be increased by adding more insulation. For outer radii greater than the critical value an increase in insulation thickness will cause a decrease in heat transfer.

The results shown in Fig. 4 are related to the previous work in which the effect of radiation term was ignored (Fedorov and Shul’gin, 2006) and the results in Fig. 3 demonstrate the impact of this term in ignition of magnesium particle temperature. As seen, the highest temperature is achieved when the radiation term is considered in energy conservation equation and the lowest temperature is achieved without radiation term. Differences between values of gas temperature in Fig. 4 and Fig. 3 are also considerable.

In the previous model, heat transfer mechanism was just by conduction, however in this model, the conduction heat transfer and the heat transfers from surrounding gas radiation to particle is investigated.

Since the heat radiation is transferred from the surrounding gas to the gas and particle fuel, the gas and particle temperature increase due to the energy absorption. So most amount of energy transfers to the particle surface, and it is obvious that the temperature of the particle surface increases, due to gaining more heat, and physically this phenomenon is acceptable.

The investigation of variation of particle temperature versus the solid particle radius ($\bar{r}$), with various surrounding film thicknesses in the planar case shows that the influence of the radius of the metal partial on the variation of particle temperature appears to be weak for different values of $L$. On the other hand we can assume that the particle temperature is approximately constant.

The non-dimensional gas phase temperature as a function of non-dimensional radius for values of non-dimensional gas layer thickness in planar case is shown in Fig.5; it can be noted that increasing the value of the radius results in a decrease of the gas phase temperature.

In Fig. 6, the value of the particle temperature in branch I (regular heating regime), is plotted as a function of $\bar{r}$ for given values of $L$ in case of spherical coordinate.
In Fig. 7 the calculated gas temperature $\overline{T}_g$ for Aluminium particle combustion is plotted as a function of the particle radius, $r$; it shows that for given value of $r$ the gas phase temperature gradient decreases with decreasing values of $\overline{L}$.

The gas phase temperature profile of heating of iron particle is plotted against the sample radius $r$ in Fig.8. In accordance with the condition in case of aluminium and magnesium particles, the gas temperature gradient decreases with the decrease of surrounding film thickness.

VI. CONCLUSIONS

In this paper, an analytical model was presented to investigate the effect of radiation in the combustion of fine metal particles, in both planar and spherical coordinates. Also the mathematical model with allowance for the heterogeneous chemical reaction and the region of the thermal influence of particle on gas is developed to describe the thermal behaviour of a magnesium particle in a gas medium.

Considering the complications in the modeling of radiation term in the combustion of fine particles, some simplifications were made in order to find the influence of radiation on temperature profile of gas and particle phase.

By solving the energy equation in each phase, and equating the heat fluxes and temperatures at one side of the boundaries with their equivalents at the other side, we can obtain an algebraic relation for temperature and draw temperature profiles of gas and particle phases.

Due to the addition of the term of radiative heat transfer to the particle and gas phase energy conservation equations, the influence of radiation on temperature profiles was found to be an increase in temperature of the gas and particle phase as strong and weak respectively.

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