MAXIMUM ALLOWABLE DYNAMIC PAYLOAD FOR FLEXIBLE MOBILE ROBOTIC MANIPULATORS

M.H. KORAYEM†, H.N. RAHIMI‡, A. NIKOOBIN§ and M. NAZEMIZADEH†

† Robotic Research Lab, School of Mechanical Engineering, Iran University of Science and Technology, Tehran, Iran. hkorayem@iust.ac.ir
‡ Department of Mechanical Engineering, Damavand Branch, Islamic Azad University, Damavand, Iran. hamedrahimi.n@gmail.com
§ Department of Mechanical Engineering, Semnan University, Semnan, Iran. anikoobin@iust.ac.ir

Abstract—Finding the full load motion for a point-to-point task can maximize the productivity and economic usage of the mobile manipulators. The presented paper proposes a technique to determine the maximum allowable dynamic payload for flexible mobile manipulators along with the obtained optimal trajectories. Non-linear modeling of the mobile robotic manipulators by considering both link and joint flexibility is presented; then, optimal motion planning of the system is organized as an optimal control formulation. By employing indirect solution of the problem, optimal maximum payload path of such robots is designed for a general objective function. The paper specially focuses on effects of various important parameters on the maximum payload determination and analyzes them thoroughly. The effectiveness and capability of the proposed method is investigated through various simulation studies. The obtained results illustrate the influences of the performance index, operating time and robot characteristics on the maximum payload path.

Keywords—Flexible Manipulator, Mobile Robot, Optimal Control, Maximum Payload.

I. INTRODUCTION

Using of lightweight robotic manipulators has been increased in the wide areas, for example manufacturing automation, construction, military and space exploration. Besides the advantages of flexible manipulators such as reducing the energy consumption and safer operation due to reduce inertia, the use of such systems has been recognized as a possible solution to increase the load-to-mass ratio and enhance payload capacity. The first formulation to obtain the maximum payload of a manipulator in the point to point motion was presented by Wang and Ravani (1988). They used the iterative linear programming (ILP) method to solve the problem. But, on their analysis the manipulator links were assumed to be rigid. Tu and Rastegar (1994) investigated the effects of payload on the dynamics of a single link manipulator with the flexible link. Also, the effects of payload on the vibration excitation of a 2R robot manipulator were studied by Parks and Pak (1991). Yue et al. (2001) used the finite element method for describing the dynamics of the kinematically redundant flexible manipulators. Then, they computed the maximum dynamic payload trajectory for the flexible robot manipulators.

Mobile manipulators are combined systems consists of a robotic manipulator mounted on a mobile platform. These systems are able to accomplish complicated tasks in large workspaces. They have a compact structure and high maneuverability and are cost effective. Planning a point-to-point task for mobile manipulators has been an important problem that has given rise to much attention. In addition, finding the full load motion on the obtained optimal trajectory in mobile manipulators can maximize the productivity and economic usage of the systems. Finding the maximum payload and corresponding optimal path was formulated as a trajectory optimization problem by Korayem et al. (2009). Tanner (2003), by implementing a potential field technique using point-world topology proposed a methodology to motion planning for multiple mobile manipulators. An appropriate mapping was employed to reduce the order of differentials of non-holonomic constraints of mobile manipulators by Papadopoulos and Pouliakakis (2001). Then, the path was designed via polynomial functions. A comprehensive literature survey on non-holonomic systems was demonstrated by Bloch (2003).

In particular, the greatest disadvantage of the mobile robotic manipulators is that most of these systems are powered on board with limited capacity. So, incorporating light links can minimize the inertia and gravity effects on links and actuators, and it results to decrease the energy consumption in the same motion. The review of the literature shows that limited research has been carried out for modeling and control of the flexible manipulator mounted on the mobile platform (Modi and Chan, 1991). Moreover, because of the fact that synthesized of flexible character of the links and mobility of the base complicates the dynamics of the system and presents the challenge in the optimization problem, in most cases, link flexibility is neglected in the simulation procedure (Gariblu and Korayem, 2006). Another important aspect in the modeling of the flexible robotic manipulators is considering the flexibility of joints in the robot dynamic equations. However, despite of the fact that it has been determined experimentally that joint flexibility exists in most manipulators, limited researches has been reported on modeling the both link and joint flexibility (Rahimi et al., 2009).

In this paper the non-linear modeling and control of the mobile robotic manipulators by considering both links and joints flexibility have been studied. The paper
specially focuses on the maximum payload determination of such robots. Paper first analyzes the dynamic model of flexible mobile robot manipulators, and then proposes a technique for optimal trajectory planning of such robots. After that, maximum allowable dynamic payload has been determined along with the obtained optimal trajectory in the various simulations. The approach is based on indirect solution of optimal control problem defined by Pontryagin. With respect to most trajectories optimization techniques found in the scientific literature, the method described in this work enables to overcome the high nonlinearity nature of the dynamic model and optimization problem. Moreover, in this method boundary conditions are satisfied exactly, while the results obtained by methods such as ILP have a considerable error in the final time. Other advantage of the proposed method is obtaining the maximum payload trajectory for each considered objective function. It means that the maximum payload and corresponding trajectory can be achieved with respect to the wide range of parameters.

II. DYNAMIC MODELING AND PROBLEM FORMULATION

A. Dynamic Modeling
In this section, a mathematical model of a general flexible links/joints mobile manipulator is developed. First, the Lagrangian assumed modes formulation is employed to derive the dynamic equations of flexible link mobile manipulators:

\[ M(Q, \dot{Q}) + H(Q, \dot{Q}, \ddot{Q}) = T, \]

where \( M \) is the inertia matrix, \( H \) is the vector of Coriolis and centrifugal forces in addition to the gravity effects vector and \( T \) is the vector of actuator efforts. The generalized coordinates of the system \( Q \) is consisted of two parts; first generalized coordinates defined rigid body motion of links \( \ddot{q}_i = (q_1, q_2, ..., q_n)^T \) and second, the generalized coordinates that related to the flexibility of the links

\[ \ddot{q}_i = (q_{11}, q_{12}, ..., q_{n1}, q_{12}, ..., q_{n2}, ..., q_{nm})^T, \]

where \( n \) and \( n_f \) are number of links and manipulator mode shapes, respectively. So one can write

\[ Q_1 = [q_1^T | q_i^T] \]  

(2)

To include the dynamic of flexible joints in the system dynamic equations, the link positions are let to be in the state vector as in the case with rigid manipulators. Actuator positions must also be considered because in contradiction to rigid robots these are related to the link positions through the dynamics of the flexible elements. By defining the link number of the flexible joints manipulator as \( n \), position of the \( i^{th} \) link is shown with \( \theta_i \): \( i=1,2, ..., n \) and the position of the \( i^{th} \) actuator with \( \theta_{ni} \): \( i=1,2, ..., n \). Now, modeling the elastic mechanical coupling between the \( i^{th} \) joint and link as a linear torsional spring with constant stiffness coefficient \( k_i \) leads to develop the Eq. (1) for the flexible link/joints mobile manipulators:

\[ M\ddot{Q}_1 + H(Q_1, \dot{Q}_1, \ddot{Q}_1) = T, \]

where \( K = [k_1, k_2, ..., k_n] \) and \( Q_1 = [Q_1, \dot{Q}_1, \ddot{Q}_1] \) is the joint torsional stiffness matrix and \( J = \text{diag}[J_1, J_2, ..., J_n] \) is the diagonal constant inertia matrix of motors. Also, \( Q_2 = [\theta_1, \theta_2, ..., \theta_n] \) is the generalized coordinates according to the joint flexibility. Finally, the overall generalized coordinates of the system can be rearranged as \( Q = [Q_1^T, Q_2^T]^T \).

The actuator efforts, \( T \), belong to the feasible set of \( \Omega \), are defined as

\[ \Omega = \{T | T_{\text{min}} \leq T_i \leq T_{\text{max}} ; \ i = 1, 2, ..., n\} \]

(4)

The optimization problem is to find control \( T \), so that the manipulator in Eq. (3) can carry maximum payload from an initial configuration to the final motion target.

B. Formulation of the Optimal Control Problem
By defining the states regarding to the rigid body motion of links as \( X_{1i} \), the flexibility of links as \( X_{f} \) and the joints flexibility as \( X_{j} \), system states can be obtain as \( X = [X_{1i}^T, X_{f}^T, X_{j}^T]^T \), where \( X_{1i} = [x_{1i}, x_{12}, x_{13}, ..., x_{nm}]^T \) and \( X_{f} = [x_{21}, x_{22}, x_{23}, ..., x_{nm}]^T \). Accordingly, Eq. (3) can be rewrite in state space form:

\[ X = f(X, T, t) \]

(5)

For the maximum payload determination problem the state departing from the initial conditions \( X(0) = X_0 \) must reach the final conditions \( X(T_f) = X_f \) in such a way that the maximum payload can be carried. Generating optimal movements can be achieved by minimizing a variety of quantities involving directly or not some dynamic capacities of the mechanical system. Optimal control is a natural formalism for the representation of such problems. The above problem can be expressed as follows: The basic idea to improve the formulation is to find the optimal path for a maximum payload. Hence, a performance index is expressed as some functional evaluated over the trajectory

\[ J(u) = \int_{t_0}^{t_f} L(X, T, t)dt \]

(6)

where \( L \) is a smooth and differentiable function. In this paper, the performance measure is defined as:

\[ J_q(T) = 0.5\int_{t_0}^{t_f} (T^TRT + X_{f}^TWX_{f})dt \]

(7)

where \( W \) is symmetric, positive semi-definite weighting matrix and \( R \) is symmetric, positive definite matrix. The objective function specified by Eqs. (6) and (7) is minimized over the entire duration of the motion. There may also be certain pragmatic constraints (reflecting such concerns as limited actuator torques) on the inputs. For example:

\[ \int \{T(X_{f})\} = T_{\text{max}}(X_{f}) \]

(8)

This is a fairly classical formulation of an optimal control problem in the Bolza form. According to the minimum principle of Pontryagin (Kirk, 1970), minimization of the performance criterion defined by Eq. (6) is achieved by minimizing the Hamiltonian defined as
Numerical technique are given by Shampine A. Background

In this study, BVP4C command in ods such as shooting, collocation, and finite difference problemas such as filling, collocation, and finite difference solve the problem. In this study, BVP4C command in ods such as shooting, collocation, and finite difference efficient commands for solving such non-linear problems. Numerical libraries offer numerous influential and corresponding maximum payloads are defined based on various parameters such as operating scriptions and corresponding maximum payloads are defined based on various parameters such as operating scriptions and corresponding maximum payloads are defined based on various parameters such as operating scriptions and corresponding maximum payloads are defined based on various parameters such as operating scriptions and corresponding maximum payloads are defined based on various parameters such as operating scriptions and corresponding maximum payloads are defined based on various parameters such as operating

The aforementioned equations lead to transform the problem of optimal control into a nonlinear two-point boundary value problem. Indeed, substituting the computed control equations in the state and co-state equations yields ordinary differential equations, where the functions $X(t)$ and $P(t)$ must satisfy boundary conditions. Numerical libraries offer numerous influential and efficient commands for solving such non-linear problems. These commands by employing competent methods such as shooting, collocation, and finite difference solve the problem. In this study, BVP4C command in MATLAB" which is based on the collocation method is used to solve the obtained problem. The details of this numerical technique are given by Shampine et al. (2000).

III. IMPLEMENTATION

A. Background

In this section, implementation is performed for a mobile manipulator consists of two flexible links/joints planar manipulator attached at the main axis of a differentially driven vehicle at point F. A concentrated pay-loading mass $m_b$ is connected to the second link as shown in Fig. 1.

Using Eq. 3, dynamic equations associated with the two links flexible mobile manipulator considering pinned – pinned mode shapes with one mode for each link are derived as:

$$\begin{bmatrix} m_1 & m_2 & m_3 & m_4 \ 
\ddot{\theta}_1 & \ddot{\theta}_2 & \ddot{\theta}_3 & \ddot{\theta}_4 \n \end{bmatrix} = \begin{bmatrix} k_1 & 0 & 0 & 0 \n 0 & k_2 & 0 & 0 \n 0 & 0 & k_3 & 0 \n 0 & 0 & 0 & k_4 \n \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 - \dot{\theta}_2 \ 
\dot{\theta}_2 - \dot{\theta}_3 \n \dot{\theta}_3 - \dot{\theta}_4 \n \dot{\theta}_4 \n \end{bmatrix} \text{= 0} \quad (13)$$

where, $e_i$ is the mode shape function. Now, the set of state space equations of the system can be defined, using Eq. 5, and the system state vectors as:

$$X_{1} = \begin{bmatrix} \dot{\theta}_1(t) \\
\dot{\theta}_2(t) \n \end{bmatrix}, \quad X_{2} = \begin{bmatrix} \dot{\theta}_3(t) \\
\dot{\theta}_4(t) \n \end{bmatrix}, \quad X_{3} = \begin{bmatrix} \dot{x}_1(t) \\
\dot{x}_2(t) \n \end{bmatrix}, \quad X_{4} = \begin{bmatrix} \dot{x}_3(t) \\
\dot{x}_4(t) \n \end{bmatrix} \text{= 0} \quad (14)$$

The three extra Dofs arose from the base mobility are solved using the augmented Jacobian matrix, and the constraint for rolling without slipping condition (Korayem et al., 2009),

$$\dot{x}_r \sin(\theta_b) - \dot{y}_r \cos(\theta_b) + L_0 \dot{\theta}_b = 0, \quad (15)$$

where $L_0$ is the distance between the junction between the first joint (in point $F(x_1, y_1)$) and the center of the base as shown in Fig. 1. Also, in all simulations the mobile base is initially at start point $(x_{fi} = 0.5m, y_{fi} = 0.5m, \theta_{fi} = 0)$ and moves along a straight-line path to final position $(x_{fi} = 1.5m, y_{fi} = 1m)$. The necessary parameters used in simulations are given in Table 1.

Velocity at start and stop point is considered to be zero. Other boundary conditions are assumed to be:

$$x_i(0) = x_i(0) = 120^\circ, \quad x_{f1}(0) = x_{f1}(0) = 90^\circ; \quad (16)$$

$$x_{f1}(f) = x_{f1}(f) = 30^\circ, \quad x_{f2}(f) = x_{f2}(f) = 30^\circ; \quad (16)$$

B. Simulations

In this section, various numerical simulations are presented based on the mathematical model describes in the section 2. In these simulations, the robot optimal trajectories and corresponding maximum payloads are defined based on various parameters such as operating time, robot characteristics and the problem performance index. Note that in all simulations, the payload is calculated with the accuracy of 0.1 Kg. Also, in all simulations the performance index is defined for minimizing the actuating inputs in addition bounding the joint velocities. For this reason, it form as

<table>
<thead>
<tr>
<th>Table 1. System parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Properties</td>
</tr>
<tr>
<td>Length of Links</td>
</tr>
<tr>
<td>Mass Density</td>
</tr>
<tr>
<td>Flexural Rigidity</td>
</tr>
<tr>
<td>Max. no Load Speed of Motor</td>
</tr>
<tr>
<td>Motor Stall Torque</td>
</tr>
<tr>
<td>Moment of Inertia (Motor)</td>
</tr>
<tr>
<td>Spring Constant</td>
</tr>
</tbody>
</table>
Then, by considering the co-state vector as \( \mathbf{P} = [p_1, p_2, \ldots, p_{12}] = [x_{13}, x_{14}, \ldots, x_{24}] \), the Hamiltonian function can be expressed as:

\[
H = \frac{1}{2} \left( r_1 t_1^2 + r_2 t_2^2 + \sum_{i=1}^{6} w_i x_i^2 \right) + \sum_{i=12}^{12} x_{12} x_i
\]

In all simulations, the penalty matrix of control efforts \( R_{2 \times 2} \) is assumed to be \( R = \text{diag}[0.01] \). The control equations can be computed using Eq. (11). The actuators which are commonly used for medium and small size manipulators are the permanent magnet D.C. motor. The torque speed characteristic of such D.C. motors may be represented by the following linear equation:

\[
T^* = K_i - K_s X_1, \quad T^* = -K_i - K_s X_2
\]

where \( K_i = [\tau_s, \tau_s]^T \), \( K_s = \text{diag}[(\tau_s / \omega_m) \tau_s / \omega_m] \), \( \tau_s \) is the stall torque and \( \omega_m \) is the maximum no load speed of the motor.

**Simulation A**

In the first case, effects of changing in performance index in the path planning problem are investigated. The payload is assumed to be 1 Kg and simulation is done for the different values of \( W = (w, w, 0, 0, w, w) \) and optimal paths for a given payload are obtained. Note that in this vector, \( w \) refers to the velocities of joints and actuators and zero refers to the velocities of mode shapes. The simulations are performed for different values of \( w \) as 0, 1, 100 and 1000.

Figure 2 shows the angular velocities of joints. The computed torques are plotted in Fig. 3. As shown in these figures increasing \( w \) leads to reduce the maximum velocity magnitude while the torqueses are growing. This issue is predictable, since increasing \( w \) in Eq. (7) leads to rise of the penalty on velocity and it decreases the penalty on torque. Furthermore, it can be found from figures, in order to attain a smoother path with smaller amount of velocity, more efforts must be applied. Also, it is obvious that all the obtained graphs are satisfied the system cost function in Eq. (7), hence, they specify optimal trajectories of the system motion. Therefore, in the proposed method designer is able to choose the most appropriate path among the various obtained optimal paths by choosing the proper penalty matrixes.

**Simulation B**

In this case \( w \) is assumed to be constant at \( w = 1 \). The robot path planning problem is investigated by increasing the payload mass until the maximum allowable payload be determined. The maximum payload is obtained as \( m_p = 8.4 \) kg (case 4). The obtained angular positions, angular velocities and torque curves graphs for a range of \( m_p \) given in Table 2, are shown in Figs. 4-6. It can be found that increasing the \( m_p \) results to enlarge the velocity values. Also, as shown in figures, increasing the payload increases the required torque until the maximum payload obtained. So that for the last case the torque curves lay on their limits. Hence, it is the most possible values of the torques and increasing the payload is impossible, because it leads to violate the boundary conditions.

![Fig. 2 - Angular velocities of joints with respect to time](image)

![Fig. 3- Torques of motors with respect to time.](image)
Fig. 4- Angular positions of joints with respect to time.

Fig. 5- Angular velocities of joints with respect to time

Fig. 6 - Torques of motors with respect to time

Table 2. The values of $m_p$ used in the simulation.

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_p$ (kg)</td>
<td>1</td>
<td>3</td>
<td>7</td>
<td>8.4</td>
</tr>
</tbody>
</table>

Table 3. The values of $w$ and corresponding calculated maximum payloads

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W$ (kg)</td>
<td>1</td>
<td>400</td>
<td>600</td>
<td>800</td>
</tr>
<tr>
<td>$m_{p_{max}}$ (kg)</td>
<td>8.4</td>
<td>7.9</td>
<td>7.5</td>
<td>6.3</td>
</tr>
</tbody>
</table>

$m_{p_{max}} = 8.4$ kg is the maximum allowable payload for the selected penalty matrices while choosing other penalty matrices, results in other optimal maximum payload trajectories. To demonstrate that issue, simulations are carried out for different values of $w$ given in the next section.

Simulation C

In this case, the maximum payload of flexible mobile manipulator is calculated and corresponding optimal trajectory at point-to-point motion is illustrated for different values of $w$ given in Table 3. The computed torques for these cases are plotted in Fig. 7. As it can be seen, increasing $w$ causes to increase oscillatory behaviors of the systems that results to reduce the maximum dynamic payload as shown in Table 3.

Simulation D

The effect of time on the calculating of maximum payload is investigated in this section. $w$ is fixed at $w=1$. The range of times and corresponding calculated maximum payload are given in Table 4. It can be concluded from table, raising the execution time leads to considerable increase of the obtained maximum payload value. The reason of this issue is that by increasing the execution time the acceleration and velocity of the system are reduced. Hence, the centrifugal and Coriolis forces which applied to the system are decreased and it causes to increase the maximum payload.
Simulation E

In these simulations the effects of links and joints flexibilities on the maximum payload is investigated. w is fixed at w=1. First, in order to investigate the roll of the link flexibility on the maximum payload value, simulation is performed for various link stiffness while joint stiffness is remained constant at K=1000 N/m. The link stiffness values (EI), and the corresponding maximum payloads are given in Table 5. It is obvious that, increasing the link rigidity causes to increase the value of the maximum payload. Amplitudes of mode shape vibrations and torques of motors with respect to time are depicted in Figs. 8 and 9. As it can be seen from figures, oscillatory behavior of the robot is increased when EI is reduced. So, the link flexibility affects the robot vibrations significantly.

Another simulation is carried out to investigate the effect of joint stiffness in the obtained maximum payload of the systems. Simulation is performed for a flexible joint manipulator with different values of joint stiffness while EI is remained constant at EI=100 kg.m^3/s^2. The values of spring constants, K, and the obtained maximum payloads are given in Table 6. It is observed that by increasing the joint stiffness value, the maximum payload is increased. Angular positions and velocities of links and motors in the first case are given in Figs. 10 and 11. It shows that both the link angular positions and velocities have deviations from their respective motor angular positions and velocities. It can be concluded that link and joint flexibility significantly affect on the system oscillation and robot behavior. Hence neglecting of such important affects in modeling and control of robots, results in the considerable and irreparable errors in the applicable usage.

IV. CONCLUSIONS

In this paper, formulation of planning the maximum payload trajectory for the flexible links/joints mobile manipulator in point-to-point motion has been presented. First, flexible mobile robot manipulators has been modeled, and then a technique for optimal trajectory planning of such robots has been proposed based on the indirect solution of the optimal control problem.
After that, maximum allowable dynamic payload has been determined along with the obtained optimal trajectories in the various simulations. The obtained results illustrate the influences of the problem performance index, operating time and robot characteristics on the maximum payload trajectory. As a result, using the proposed method, the designer can choose the most appropriate path among the various optimal paths according to designing necessities. Also, the optimal maximum payload trajectory obtained using this method can be used as a reference signal and feed forward command in the closed-loop control of the robot manipulators.

REFERENCES


Received: November 27, 2011.
Accepted: April 16, 2012.
Recommended by Subject Editor Jorge Solsona.