AN EFFICIENT SPECTRAL SHAPING METHOD FOR OFDM SYSTEMS USING CORRELATED INTERPOLATION OF SYMBOLS

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Abstract—The generation of suitable orthogonal frequency division multiplexing (OFDM) signals on the grounds of fully digital signal processing is considered. The main objective is to obtain a discrete-time signal with adequate allocation of power emissions in both, in-band portion (i.e., the allocated band for communication) and out-of-band portion (i.e., the band allocated to adjacent channels) of the spectrum. The proposed method prevents the transmitter from using traditional filtering techniques, to keep under control power emissions in the system. In addition, the adaptability feature of our proposal makes its implementation attractive within cognitive radio (CR) and software defined radio (SDR) OFDM-based systems. Our spectral shaping approach is based on an optimum interpolation, obtained from the combination of an inverse fast Fourier transform (IFFT) and a spectral precoding operation, both of them transparent from the perspective of a conventional (legacy) OFDM receiver.

Keywords—Correlated interpolation, IFFT, double-length, OFDM, N-continuous, symbol merging, spectral precoding, spectral shaping.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is an attractive modulation technique that is not only widely used in current commercial systems (e.g., xDSL, DVB, WiFi, and WiMAX), but it also represents the selected candidate to be implemented in the air interface of future high-speed mobile communications standards (i.e., LTE and LTE-Advanced). The available spectrum in an OFDM transmission is occupied by orthogonal subcarriers, which are utilized to convey parallel data streams across non-interfering portions of the channel. The use of OFDM allows to exploit effectively both, the frequency- and time- domain dimensions of a (slowly varying) frequency selective fading channel, as it admits simple single-tap equalization using a cyclic prefix (CP) (Hwang et al., 2009). Another important property is the possibility of implementing the required transmit signal processing operations completely in the digital domain, using the well-known and efficient fast Fourier transform (FFT) algorithm. Nevertheless, the implementation of OFDM in practice comes with many challenges. For example, high levels of out-of-band power emissions may be generated in transmission (due to discontinuities in the time-domain OFDM signal) if no corrective measures are taken. Trying to alleviate this problem, in this paper we focus on the use of a novel interpolation method that we named correlated interpolation (CI), as a simple way to achieve a suitable spectrum shape of the output signal in a practical system implementation.

An OFDM signal is a sequence of OFDM symbols, each one consisting of a collection of modulated orthogonal subcarriers. Since the amplitudes and phases of the subcarriers are often statistically independent, OFDM symbols are considered independent as well. Due to this phenomenon, the concatenation of OFDM symbols introduces discontinuities in the corresponding time-domain signal, or equivalently, high levels of out-of-band power emissions are generated. The simplest solution to this problem consists in using filtering techniques that limit this undesired power leakage out-of-band; nevertheless, the main drawback of such an approach is a reduction in the effectiveness of the CP that is introduced (van de Beek and Berggren, 2009a). Mahmoud and Arslan (2008) proposed an interesting alternative to control this problem, which is basically based on implementing adaptive symbol transitions when generating the OFDM signal. However, the main drawback in this case is the necessity to update the transmitter signal processing on a per-symbol basis, increasing as a consequence the complexity of the system (i.e., its implementation requirements).

This work focus on the generation of OFDM signals that comply with common quality requirements specified by emission masks, such as the ones that are presented in LTE specifications (3GPP 36.211, 2012). In other words, we address the generation of an oversampled digital signal that improves the spectral allocation of power in both, in-band and out-of-band regions. The key idea behind our proposal is based on correlating two OFDM symbols in frequency-domain, to obtain a continuous behavior in time-domain by means of the CI. This is obtained using an inverse discrete Fourier transform (IDFT) representation with double length, when compared to the one that is used in conventional OFDM transmissions. At that time, the concatenation points in the resulting sequence of OFDM symbols will show a continuous behavior in half of the original merging points (that correspond to the whole OFDM transmission). A preliminary version of this merging concept was previously presented in Lizarraga et al. (2011a).

A spectral precoding technique is additionally introduced in this paper, with the ultimate goal of guaranteeing a continuous time-domain behavior in the remaining
concatenation points of the OFDM signal (i.e., in those time instants where the previously introduced CI processing is not able to provide a smooth transition). Unlike the scheme given by Mahmoud and Arslan (2008), this procedure consists of a static precoding, and does not necessitate extra spectral resources for its implementation like in alternative proposals (e.g., Chen et al., 2011). Simulation results show an important reduction in the out-of-band power emissions, when compared to alternatives previously reported in the literature (e.g., van de Beek and Berggren, 2009b). Moreover, a lower distortion in frequency domain is observed, when using the error-vector magnitude (EVM) as performance measure to assess this effect.

The rest of this paper is organized as follows. Section II presents the OFDM signal equations that are used throughout this work. The fundamentals of both, CI scheme and reduced distortion precoding proposal are presented in Section III and Section IV, respectively. Section V covers the channel modeling and the system operation at the receiver side, while simulation results are given in Section VI. Finally, Section VII shows the main conclusions.

Notation: Vectors are indicated in bold and lower-case letters, unlike complex or real scalar numbers. Matrices are indicated in bold with capital letters. Superscripts \( t \), \( H \), \(-1\) represent transpose, Hermitian, and inversion, respectively. Notation \( \mathbf{0}_{m,n} \) indicates the \( M \times N \) matrix with all-zero entries, while \( I_M \) stands for the \( M \times M \) identity matrix.

II. CONVENTIONAL SIGNAL MODELING

Let us define \( s(t) \) as a continuous time OFDM signal composed by the concatenation of successive OFDM symbols \( s_i(t) \), i.e.,

\[
s(t) = \sum_{i} s_i(t-t_i) .
\]

Thus, the \( i \)-th symbol \( s_i(t) \) has a duration \( T_c \), and derives from the sum of \( K \) orthogonal subcarriers belonging to the so-called subcarrier allocation scheme \( \mathcal{K} \), i.e.,

\[
s_i(t) = \sum_{k=0}^{K-1} d_{i,k} p_{i,k}(t) = \mathbf{p}^T(t) \mathbf{d}_i .
\]

The amplitude and phase of each subcarrier is determined by the \( k \)-th complex symbol \( d_{i,k} \) (obtained from a prior complex modulator). Complex symbols are grouped into the \( K \)-element column vector \( \mathbf{d}_i \). The \( K \)-element column vector \( \mathbf{p}(t) \) contains at each entry the corresponding subcarrier symbol \( p_{i,k}(t) = e^{j2\pi f_k t} \) for \( t \in T \).

The interval \( T \) is defined as \( [-T_p, T_p] \), where \( T_p \) and \( T_s \) represent the duration of the CP and the symbol period, respectively (with \( T = T_p + T_s \)). Note that the total symbol rate is determined by the relation \( K/T \).

A sampled version of the \( i \)-th symbol without CP can be written as \( s_i = (1/K)F^T \mathbf{d}_i \), where \( F = \{F_{k,b}\} \) represents the \( K \)-point discrete Fourier transform (DFT) by means of a matrix with elements \( F_{k,b} = \exp(-j2\pi k b/K) \), with both \( k,b = 0, \ldots, K-1 \). Then, this matrix-vector approach can be with elements replaced by an inverse fast Fourier transform (IFFT) calculation (to take advantage of well-known efficient algorithms), i.e.,

\[
s_i(t) = \text{IFFT} \{d_i\}, \quad \mathcal{K} = \{0,1, \ldots, K-1\} \bigcup \{K \} ,
\]

where IFFT(\( a,b \)) represents the IFFT calculation with \( b \) points applied on the \( a \) vector. As stated by balanced subcarrier allocation schemes defined in several standards, we consider the case

\[
\mathcal{K} = \{-K/2, \ldots, -1, 0, \ldots, K/2\} ,
\]

which is included in 3GPP 36.104 (2012). To comply with this subcarrier mapping, \( u^{x} = \text{CSh}(d^{x}, K/2) \) can be applied as argument of the IFFT operation, considering \( d^{x} = (d_{0,0}, \ldots, d_{1,K-2}, 0, \ldots, d_{K-1,K-2}) \) and assuming that CSH(\( a,b \)) represents a circular up-shift operation of \( b \) positions in the column vector \( a \). This allows us to get the output signal

\[
s^{x} = \text{IFFT}(u^{x}, K+1) ,
\]

which is an expression equivalent to (3), but considering (4).

In applications such as software defined radio (SDR) and cognitive radio (CR) (Paweleczak et al., 2011), the bandwidth that is considered to generate the output signal is allowed to be larger than the bandwidth that is assigned to the in-band channel. The aim in this case is to have the power emissions adequately accommodated within certain (and possibly variable) emission mask. To achieve this goal, an oversampled discrete-time output signal is required. Oversampling is related to interpolation, and it does not demand extra bandwidth resources for communication purposes. Let us define \( \eta \) as the oversampling factor. Then, following a similar approach to the one presented in (5), we are able to obtain an oversampled discrete-time output signal from

\[
s^{\eta x} = \eta \text{IFFT}(u^{x}, (K+1)\eta) ,
\]

where \( u^{x} \) can be seen as a conventional zero-padded version of \( u^{x} \), defined by

\[
u^{x} = \left[u_{0}^{x}, u_{1}^{x}, \ldots, u_{K/2}^{x}, 0, u_{K/2+1}^{x}, \ldots, u_{K}^{x}\right]^T
\]

At this stage, \( s^{\eta x} \) attains the form of a vector with \( (K+1)\eta \) entries, containing the oversampled output signal. The classical CP insertion can be performed by the extension \( s^{\eta x, CP} = (s^{x, CP}_{\eta})^\top \), where \( u = T_g/T_s \) represents the CP fraction (i.e., the guard interval fraction). An OFDM symbol with CP is then obtained by means of concatenation, and is given by

\[
s^{\eta x, CP} = (s^{x, CP}_{\eta})^\top .
\]

The corresponding non-oversampled expression (i.e., \( s^{x, CP} \)) results when setting \( \eta = 1 \). Here, we emphasize that the concatenation between the original signal and its cyclic extension generates a continuous signal at the merging point. This follows from the cyclo-stationary property of the IDFT, which shows continuity between the first and last point of the Fourier anti-transformed signal (see Fig. 1 in Lizarraga et al., 2011a).

III. CORRELATED INTERPOLATION

The IFFT calculation of a conveniently zero-padded vector produces a continuous interpolated signal, which
still shows a smooth transition between the first and the last point. Based on this observation, the avoidance of discontinuities using a convenient IFFT processing when generating a sequence of OFDM symbols arose for first time. As previously stated, our goal is to exploit this property to reduce out-of-band power emissions in an OFDM-based system.

In the first instance we consider an OFDM signal that has no cyclic prefix, by setting the value $\nu=0$. In this situation, it is valid to take into account the concatenation point between successive symbols with indexes $i$ and $i+1$, for even values in $i$. Then, our proposal is based on obtaining a discrete-time signal that is equivalent to the one presented in (5) for $s^x_i$, and that is jointly obtained from a single IFFT calculation that uses a double number of points. To achieve this representation, we start by defining

$$u^{x^+}_i = \frac{1}{\sqrt{k+1}} \left( u^x_i + u^x_{i+1} \right), \quad (8)$$

$$u^{x^-}_i = \frac{1}{\sqrt{k+1}} \mathbf{F} \Psi \mathbf{F}^H (u^x_i - u^x_{i+1})$$

where $\Psi = \text{diag}(e^{-j\pi x/k}, e^{-j2\pi x/k}, \ldots, e^{-j(k-1)\pi x/k})$, and $\mathbf{F}$ represents the corresponding $(K+1)$-point DFT matrix. Then, we define the interlacing matrix

$$\mathbf{E} = \{E_{i,m}\}$$

$$\begin{align*}
E_{i,m} & = \begin{cases}
1, & i = 0, 2, \ldots, 2K, m = 0, \ldots, 2K + 1; \\
0, & i = 1, 3, \ldots, 2K + 1, m = 0, \ldots, 2K + 1;
\end{cases} \quad (9)
\end{align*}$$

to produce the vector $u^{x^+} = \mathbf{E} \left( u^{x^+}_0, u^{x^-}_0, u^{x^+}_1, u^{x^-}_1, \ldots \right)^T$ that contains $(K+1)$ entries. With these definitions it is verified that

$$s^{x^+} = (s^{x^+}_0, s^{x^-}_0, u^{x^+}_0, u^{x^-}_0, u^{x^+}_1, u^{x^-}_1, u^{x^+}_2, u^{x^-}_2, \ldots)^T$$

If we now focus on the corresponding interpolated representation, the expression $s^{x^+} \approx \text{IFFT}(u^{x^+},2(K+1)+y)$ originates from

$$u^{x^+} = \left( u^x_0, \ldots, u^x_{2K}, 0, u^{x^+}_0, u^{x^-}_0, u^{x^+}_1, u^{x^-}_1, \ldots, u^{x^+}_{2K}, u^{x^-}_{2K} \right)^T$$

which is a conventional zero-padded version of $u^{x^+}$. Consequently, the analyzed concatenation points follow a continuous signal. Notice that some latency has been introduced in this approach. Since the OFDM symbol $s^{x^+}$ has a double length, when assuming that the bandwidth remains unchanged (or equivalently, that the sampling rate in the receiver is not modified), we obtain that the transmission interval may span two channel states. This is because the length of a conventional OFDM symbol is determined according to the stationarity of the channel. Meanwhile, if a frequency-selective fading channel is considered, the insertion of CP is necessary (i.e., $\nu > 0$ should be used). Given the equivalence in (10), it is possible to insert prefixes for $s^{x^+}$ and $s^{x^-}$ in the conventional manner. However, even though the insertion of a prefix in $s^{x^+}$ does not affect the characteristics of our proposed scheme, the insertion of a prefix before $s^{x^-}$ does alter the continuous behavior that was achieved in $s^{x^-}$. To address this problem, we define a cyclic suffix (CS) that is attached to $s^{x^-}$.

Because considering the double-length interpolation. Since both, CP and CS must be extensions extracted from the single-length symbols, it is possible to show that an extension extracted from $s^{x^+}$ implies the concatenation of signals that have no continuity. To overcome this difficulty, we consider the interpolation difference, first introduced by Lizarraga et al. (2011a). It is possible to observe that at the time instants closer to the middle part of the single-length periods, both interpolations (i.e., the double-length and single-length interpolations) are similar. With these results, it is possible to express the output signal as

$$s^{CI}_i = \left( s^{x^+}_{ix}, e^{j2\pi x(k+1), j2\pi x(k+1)}, \ldots, s^{x^-}_{ix}, e^{j2\pi x(k+1), j2\pi x(k+1)} \right). \quad (12)$$

In this construction, the cyclic extensions are already included to deal with the channel dispersion in time-domain. Later on, we will discuss the effect that these modifications have in the operation of a conventional (i.e., legacy) OFDM receiver. A block diagram of the proposed transmitter is presented in Fig. 1. In the following section, a complementary proposal based on spectral precoding is presented. Spectral precoding can be used in conjunction with CI, to further improve the performance of the OFDM-based system in the frequency-domain.

### IV. SPECTRAL PRECODING

According to the results presented in Lizarraga et al. (2011a), b), it is verified that the previous development fosters a performance improvement in terms of out-of-band power emissions. This benefit appears due to the avoidance of discontinuities between consecutive OFDM symbols, with indexes $i$ and $i+1$ for $i$ even. After this, the interest is transferred to the rest of the concatenation points of the OFDM symbol sequence, that have indexes $i$ and $i+1$ for $i$ odd (i.e., the concatenation between successive double-length OFDM symbols). Taking into account this situation, we are now ready to develop a precoding scheme that avoids the discontinuities that (might) still exist in the output signal. However, it is important to emphasize that the application of the CI can improve the performance of many other proposals for OFDM system optimization. Therefore, this approach provides a general method that can be applied for other purposes, beyond the out-of-band power emission involved here.

![Figure 1: Block diagram of the proposed OFDM transmitter.](image)

The correlation block supports the double-length symbols. Spectral precoding is applied to obtain additional improvement in the performance of the spectral shaping method.
Our development follows a similar reasoning to the one presented by van de Beek and Berggren (2009b), and considers to have a continuous signal in the concatenation of two OFDM symbols with indexes $i$ and $i+1$ for odd. When the signal and its first $N$ derivatives are considered, the idea is to verify

$$\frac{d^n}{dt^n} s_i(t) \bigg|_{t_{i-1}}^{t_i} = \frac{d^n}{dt^n} s_{i+1}(t) \bigg|_{t_{i-1}}^{t_i}$$

(13)

for $n = 0, \ldots, N$. Then, a precoder can be designed using a non-injective linear transformation which, however, allows an acceptable decoding complexity. To express the stated condition in matrix form, we introduce the row vector $p^i = \{p_j^i\}$ with entries $\{-K/2, \ldots, K/2\}$, and a matrix with dimension $(N+1) \times (K+1)$, i.e.,

$$A = \begin{bmatrix} p^0, & p^1, & \ldots, & p^N \end{bmatrix}^T,$$

(14)

where superscripts $0, 1, \ldots, N$ in the elements indicate element-wise powers. After that, we define two $(K+1) \times (K+1)$ diagonal matrices as

$$\Phi_1 = \text{diag}(e^{j\phi_0}, e^{j\phi_1}, \ldots, e^{j\phi_N})$$

and

$$\Phi_2 = \text{diag}(e^{j\phi_{K/2}}, e^{j\phi_{K/4}}, \ldots, e^{j\phi_0})$$

(15)

with $\phi_0 = 2\pi (1-n)$ and $\phi_2 = 2\pi n$. Then, the vector $v_i = (u^x_i, u^x_{i+1})^T$ brings together two single-length frequency-domain OFDM symbols circularly rotated, and is defined for even values of $i$. Consequently, considering

$$D = \begin{bmatrix} I_{K+1} & I_{K+1} \\ I_{K+1} & -I_{K+1} \end{bmatrix},$$

(16)

the $(2N+1) \times 2(K+1)$ matrix defined by

$$B = \begin{bmatrix} \Phi_1 & 0_{K+1 \times (K+1)} & \Phi_2 \\ 0_{(K+1) \times (K+1)} & \Phi_1 \end{bmatrix} D,$$

(17)

can express the constraint in (13) as $Bv_i = 0_{2(K+1)v_i}$. This restriction holds for a pair of OFDM symbols slightly distorted in frequency domain. This distorted vector is represented as $v_i = v_i + w_i$, where the vector $w_i$ with $2(K+1)$ elements identifies the distortion term. So, the precoding matrix can be now written as

$$G = I_{2(K+1)} - B^H (BB^H)^{-1} B.$$

(18)

This matrix is used to project the non-distorted vector $v_i$ onto the null space of $B$ (i.e., $\{ x \in \mathbb{C}^{2(K+1)} | Bx = 0_{2(K+1)v_i} \}$), yielding distorted vector $v_i = Gv_i$.

A. High-Order Continuity Restrictions

The expression given for $s_i^{(1)}$ in (12) is achieved replacing two fractions of the signal $s_i^{(K)}$ by the adequate fractions of $s_j^{(K)}$ and $s_{j+1}^{(K)}$. Since the interpolations in $s_i^{(K)}$ and $s_{i+1}^{(K)}$ are not equal to the corresponding interpolations in $s_j^{(K)}$ (though they are similar as $K$ grows), the concatenation of the fractions taken from $s_i^{(K)}$ and $s_{i+1}^{(K)}$ with the fraction of $s_j^{(K)}$ does not guarantee a strictly continuous behavior in the signal. However, due to the aforementioned similarities, the discontinuities that were found do not have large magnitude.

Meanwhile, it has been observed that by specifying continuity orders greater than zero in the precoder de-

sign (i.e., $N > 0$), these discontinuities must be effectively kept under control. This phenomenon can be taken into account at the precoder, defining a pair of new restrictions for time instants $t = \{1/2T_s, 3/2T_s\}$. An interesting analysis is derived from (10); based on this equality, the new specification can be omitted for $n = 0$. Additionally, simulation results justify the observation that due to interpolation similarities, these additional restrictions can be set only for $n = 1$ regardless of the value in $N$. That is, focusing solely on the first derivative constraint, we are able to control completely the out-of-band power emissions that are generated in these instants (regardless of higher order constraints that may be defined for the complete precoder). These simplifications allow to reduce the distortion in frequency domain, as well as reduce the complexity of calculating the precoding matrix $G$. Yet, this operation can be performed even offline.

In this context, we develop some additional matrices to control double-length and single-length interpolations, i.e.,

$$C = \begin{bmatrix} \text{CSH}(I_{K+1} \cdot K/2) & 0_{(K+1) \times (K+1)} \\ 0_{(K+1) \times (K+1)} & \text{CSH}(I_{K+1} \cdot K/2) \end{bmatrix},$$

(19)

$$D' = \begin{bmatrix} I_{K+1} & 0_{(K+1) \times (K+1)} \\ 0_{(K+1) \times (K+1)} & F \Psi^{-1} F^H \end{bmatrix},$$

(20)

$$C' = \text{CSH}(I_{2(K+1) \cdot (-K+1)}), \quad A' = (0, 1, \ldots, K), \quad A'' = (0, \ldots, 2(K+1),$$

and two diagonal matrices

$$\Phi_3 = \text{diag}(e^{j\phi_{K/2}}, e^{j\phi_{K/4}}, \ldots, e^{j\phi_0}),$$

$$\Phi_4 = \text{diag}(e^{j\phi_0}, e^{j\phi_1}, \ldots, e^{j\phi_N})$$

(21)

given the angles $\phi_0 = \pi K/(K+1)$ and $\phi_0 = \pi K/(2(K+1))$. Then, two new restrictions matrices can be established according to

$$\begin{bmatrix} B' = (A' \Phi_3, 0_{(K+1) \times (K+1)}) \\ B'' = (A'' \Phi_3, C' \text{ED'} C') \end{bmatrix}.$$

Finally, the precoding matrix can be computed using $\mathbf{B} = (B, B', B'')^T$, as $\mathbf{G} = I_{2(K+1)} - \mathbf{B}^H (\mathbf{BB}^H)^{-1} \mathbf{B}$.

B. Distortion Evaluation

The evaluation of our proposal considers that the decoding is carried out in conventional (legacy) OFDM receiver. Based on simulation results, we will show that no output filter will be required to keep the power emissions within pre-defined power masks (note that this is the main advantage behind our proposal). Nevertheless, the price to pay for the proposed precoding operation is a slight distortion in the frequency domain. To evaluate this phenomenon in quantitative way, the observed levels of distortion are assessed using the performance measure defined by 3GPP 36.104 (2012), i.e.

$$\text{EVM} = \sqrt{E \| F^H \mathbf{v} - \mathbf{v} \| F^H \| v \| F^H}.$$  

(23)

It is important to notice that for the precoder presented in (18), the latter expression reduces to
to highlight three observations. The first one relates to the processing introduced to achieve the signal \( s_{i}^{C1} \), which is performed at the transmitter. The second one arises from considering that this processing is transparent to the receiver, assuming conventional sampling rate and proper timing. The third observation is that the receiver must perform both, CP extraction and CS extraction, treating the received OFDM sequence containing double-length symbols in the same way as the sequence would be treated if it contains single-length symbols. Alternatively, we can focus on the operation of the receiver and analyze that for a minimum sampling rate, vector

\[
\begin{align*}
\mathbf{s}^{\text{CI-MS},1} &= (s_{K_i}^{\text{CI}-1}, \ldots, s_{K_i}^{\text{CI}}, \ldots, s_{K_i}^{\text{CI}-1}, \ldots, s_{K_i}^{\text{CI}}), \\
\mathbf{s}^{\text{CI-MS},2} &= (s_{K_i}^{\text{CI}-1}, \ldots, s_{K_i}^{\text{CI}}, \ldots, s_{K_i}^{\text{CI}-1}, \ldots, s_{K_i}^{\text{CI}}). 
\end{align*}
\]

is obtained from \( s_{i}^{\text{CI}} \). Then, we divide this vector into two parts of equal length, i.e., \( s_{i}^{\text{CI-MS}} = (s_{i}^{\text{CI-MS},1} s_{i}^{\text{CI-MS},2})^{T} \). Note that \( s_{i}^{\text{CI-MS},1} \) and \( s_{i}^{\text{CI-MS},2} \) reduce to \( s_{i}^{x} \) and \( s_{i+1}^{x} \) when eliminating the CP and CS, respectively. However, if only CP extraction is applied on both sequences, \( s_{i}^{\text{CI-MS},1} \) turns into \( s_{i}^{x} \), while \( s_{i}^{\text{CI-MS},2} \) turns into

\[
\begin{align*}
s_{i+1}^{x} &= \frac{1}{2(K+1)}F^x(Y^{-1}u_i^{x}),
\end{align*}
\]

where

\[
Y = \text{diag}(e^{-j2\pi/0}, e^{-j2\pi/1}, \ldots, e^{-j2\pi K}).
\]

An important result emerges from previous expression: the extraction of CP in symbols with annexed suffix produces a distortion in the frequency domain, which can be interpreted as a rotation in the phase of each subcarrier. Consequently, if this distortion is taken as part of the effect of the channel, we conclude that this is compensated by the equalizer of a conventional receiver. This important result justifies the use of our technique in systems with conventional receivers.

VI. SIMULATION RESULTS

The proposed system is evaluated by the aid of numerical simulations. The first analysis uses a configuration with \( K=600 \) subcarriers, allocating to each subcarrier an equal fraction of the total average transmit power (fixed to 46 [dBm]) when using 16-QAM modulation. The timing parameters are set to \( T_{s} = 1/15 [\text{ms}] \) and \( T_{c} = 9/128 T_{s} \). All these settings are in accordance to 3GPP UTRA/LTE specifications in 3GPP 36.104 (2012). The OFDM transmission is implemented in both, traditional way (i.e., using single-length symbols) and using our new proposal (i.e., implementing double-length symbols). Power spectra are estimated by means of Welch’s averaged periodogram method, with a 8192-sample Hanning window and a 1024-sample overlap for an observation time of 1 [s].

The power spectral density for each order of continuity is shown in Fig. 3. The results show a reduction in out-of-band power, keeping a constant gap in the area of adjacent channels when compared to the conventional system. It is worth noticing that this feature is manifested for both, the precoding-free scheme and the scheme with precoding for \( N=0, 1, 2, 3, 4 \). It is observed that, at 5 [MHz] from the central frequency of
the assigned band, the requirements of the power mask in presence of single-length OFDM transmission are verified using a continuity order \( N = 2 \). On the other hand, when implementing double-length OFDM transmission, the requirements of the mask are already satisfied with unitary continuity order (i.e., for \( N = 1 \)). This analysis shows that the proposed technique reduces the out-of-band power emission, at the expense of introducing a low distortion (see Fig. 2).

The reduction in out-of-band power emissions that is obtained by using CI presents a constant gap. This result allows a significant reduction in the out-of-band emissions for frequencies near the limits of the allocated band (but outside this band). This behavior is difficult to achieve by conventional filtering. In this context, we analyze the reduction gap that is observed for both, different numbers of subcarriers and various orders of continuity at the precoder. These results are shown in the Fig. 4, where gaps values in the order of 3 [dB] are observed in most cases. We note that a smaller gap was observed when the precoder targets high orders of continuity in presence of a reduced number of subcarriers (e.g., when \( N = 4 \) and \( K = 300 \)).

The performance has also been evaluated in terms of symbol error rate, in presence of additive white Gaussian noise (AWGN) channels and Rayleigh fading channels. The first case is considered by assuming a complex-valued zero-mean Gaussian noise vector with covariance matrix \( \sigma^2 I_{K+1} \), and a channel frequency response \( H_k \). In the second case, a frequency-selective fading channel is represented using a complex-valued diagonal matrix \( \mathbf{H} \), corresponding to a \( \mathbf{h} \) vector with \( s(K+1)+1 \) elements (i.e., an stochastic vector with zero-mean unit variance complex Gaussian entries).

Figure 5 shows the symbol error rate that is achieved for \( K = 600 \) by both, a conventional single-length transmission and our proposed double-length transmission with/without spectral precoding (SP). In all cases, a conventional OFDM receiver is used to decode the symbols. The degradation introduced by the precoding is mainly reflected in the error floors. Yet, the minor distortion that our proposed scheme introduces reduces this disturbance significantly in both, AWGN and Rayleigh channel cases.

To conclude, we would like to mention that the peak-to-average power ratio (PAPR) of the system is not affected when using our proposed method. This is because the double-length IFFT calculation is applied on a previously correlated vector (in frequency domain). In this manner, the obtained PAPR is equivalent to the one that is observed when analyzing two independent single-length symbols. So, the number of subcarriers that could be added to produce a peak is only \( K \), rather than \( 2K \).

Numerical simulation results were obtained to justify this interpretation; nevertheless, they are not included here since they do not provide additional insights to the main topics already discussed in this work.

VII. CONCLUSIONS

The generation of suitable orthogonal frequency division multiplexing (OFDM) signals was studied in this paper, using a practical spectrum shaping method that enables the mitigation of out-of-band power emissions. To achieve this goal, the use of a correlated interpolation (CI) of OFDM symbols was first studied. After that, the use of spectral precoding was also considered. Numerical simulations were carried out to evaluate the performance of our proposed spectrum shaping method. Important improvements have been observed in various performance measures, when compared to the performance results that are observed when using similar methods that were previously reported in the literature.

Since our method generates the OFDM signal in a completely digital way, its implementation in practice can be done robustly, without new (and usually costly) hardware modifications at the transmitter side. Due to no modifications are required at the receiver side, the same legacy OFDM processing can be used.

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Received: May 5, 2012
Accepted: October 22, 2012
Recommended by Subject Editor: Gastón Schlothauer, María Eugenia Torres and José Luis Figueroa. (special issue RPIC 2011).